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Analyzing Options Market Toxicity and the Black-Scholes Formula in the Presence of Jump Diffusion as Simulated with Agent-Based Modeling

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Abstract

This paper presents new and significant research on the Black-Scholes Formula using the agent-based modeling software NetLogo. The software was used to simulate an options market subject to jump diffusion. Since the widely-used Black-Scholes Formula has at times proven unreliable, this research sought to understand circumstances that render the formula ineffective. It was hypothesized that markets would become difficult to trade in or "toxic" at low price volatility but high jump volatility. Further, it was predicted that kurtosis would alert the presence of toxic markets by accurately and consistently conveying whether jump diffusion was present.

Keywords

Black-Scholes, Netlogo, Options market, jump diffusion, kurtosis

Cover Page Footnote

I would like to thank Dr. Samuel Kortum, Professor of Economics and Director of Undergraduate Studies of Economics at Yale University, for his review of the simulation, encouragement, and recommendation of studying the effects of jump diffusion on the model. In addition, I thank Mr. Chip Galloway, Professor of Mathematics at Collin College, for his advice on the project.

1. Introduction

1.1 Motivation

Wall Street is chaos. Money changes hands in a fraction of a second. Prices fluctuate seemingly arbitrarily, and the entire system seems little more than gambling. Yet, in recent history, a mathematical breakthrough transformed the face of the stock market. Three decades ago, Fischer Black and Myron Scholes developed the Black-Scholes Formula for "eliminating risk," so to speak [1][4]. Their enterprise earned billions of dollars and won them the Nobel Prize in Economics, as top investors would lend money in exchange for part of the return. Suddenly, the formula stopped working. The market caved, and the business started by the two mathematicians, Long-Term Capital Measurement, lost massive amounts of money in 1998. Today, the formula is still used, but with caution because of its sudden, temporary failure. The constantly shifting dynamics of the market have ensured that no one formula will stay in power for long. Many investors today use some variant of the Black-Scholes formula, though the original formula remains fairly accurate under the proper conditions.

The possibilities of risk and chaos continue to the present day. As recently as 2012, almost four years after the 2008 financial crisis, caused in part by new complex asset classes such as credit default swaps, bank JPMorgan lost two billion dollars due to continued risky trading strategies. Executives responsible for the sector of the organization that made the risky trades subsequently testified in Congress. The incident is a sobering reminder of how dangerous the options market can be when the wrong methods are used [2][3].

1.2 Options Markets

With today's complex financial instruments that affect the market, fluctuations in prices and preferences increase risk, and by doing so, often inhibit trade. People who have little knowledge of the intricacies of risk and the market can often suffer. For example, farmers may create an agreement akin to a "put option" to ensure financial security [7]. These farmers set a price to sell their year's crop, and make an agreement that, when the crop is harvested, the farmer has the option, but not the obligation, to sell his goods to the resource consumer at the previously agreed upon price, regardless of the market price of the farmer's goods at harvest time. The unpredictability of the general level of prices can harm farmers who do not know how to use this technique. If the market price of the farmer's goods caves when the farmer's crop is ready, an entire year of work could be compromised without insurance in the form of a put option.

Such precautions can be undertaken to prevent market calamities from affecting farmers and individuals in similar scenarios. Using Black-Scholes, farmers could determine appropriate prices to assign the put options for their crops. This paper presents an economic model that can be used to study how different factors and preferences play into the farmer's decision. Would it be beneficial to deviate somewhat from the formula's result? How long should one wait before entering into the market? These are the kinds of questions this research addresses. Other vendors with perishable products, such as newsvendors and bakers, can use similar techniques in their businesses. They have to price their products and predict how much to produce to satisfy customers yet minimize waste when their products expire, and maximize their business success.

The more experienced equity market traders often successfully reduce their risk in their investments. They use similar techniques as the aforementioned examples of farmers and newsvendors to reduce risk in certain investments. As a put option transfers the risk of a stock, the risk falls on whoever is willing to accept it in exchange for a chance of reward. Traders in situations where a crash on certain stocks within a certain time would be disastrous have the ability to use options as temporary insurance. By reducing the risk for their own investments, especially when making large investments, they also reduce the volatility of the market and thus the risk for everyone.

1.3 Overview of Research

The world's best-known options pricing model, the Black-Scholes Formula, is used to calculate the theoretical price of options. However, there is one important factor not incorporated into Black-Scholes which may have led to its infamous failure when used by Long Term Capital Management and in other notorious market events. The concept of jump diffusion—a model of random jumps that occur in the options market—can have significant effects on the success of traders using Black-Scholes [20]. A purely Black-Scholes model ignores possible random jumps that, while rare, can occur in the options market and render the formula unusable. The formula's difficulties in determining prices in the presence of jumps may have led to its failure in the past. In the current literature, some researchers argue that the random nature of the jumps warrants excluding the jumps from a trader's calculations, since the random walk is independent of the random jump [17]. Others believe that in a jump diffusion scenario traders should use a higher price than the Black-Scholes result to avoid losses [15]. Given the uncertainty of jump diffusion effects, this research seeks to incorporate jump diffusion into an options market model to help understand its effects. Being able to detect the presence of jumps in a given market would help protect both big and small investors and the overall economy.

This research on the Black-Scholes pricing equation uses the agent-based modeling language NetLogo, a tool used by sociologists to study group behavior, to simulate a market trading options on the price of an underlying asset behaving as a geometric Brownian motion process subject to jump diffusion. This unique approach pairs mathematical calculations with behavioral rules, simulating detailed actions and decisions in a custom program. Tracking several variables and modeling option trader decisions, the simulated agent-based options market was analyzed under varying conditions and multiple market runs. During each market run, the results of thousands of simulated traders estimating prices, each trader making hundreds of decisions, were collected, plotted, and analyzed. Successive markets were aggregated in a Monte Carlo method to converge on consistent results. This powerful, flexible model enables the user to set various parameters for the market and traders, and gives a convenient, in-depth view of key indicators of market behavior. The simulation with the inclusion of Black-Scholes and jump diffusion has been accepted into Northwestern University's NetLogo Models Library [21].

Using the NetLogo option market framework developed by the author, this research extends the model to examine the effects of jump diffusion. Based on a partial differential equations model of geometric Brownian motion representing the time series of the underlying asset price, the Black-Scholes formula calculates the current value of an option contract based on market and option parameters, including the underlying asset price volatility. However, prices sometimes experience shocks that are not accurately modeled by a geometric Brownian motion process alone. Since the Black-Scholes formula does not adequately predict these "fat tail" events, jump diffusion techniques were implemented in this research to examine the conditions under which Black-Scholes becomes unreliable. Furthermore, kurtosis, the volatility of the underlying asset price volatility, was calculated in the model, and this kurtosis was examined under jump diffusion conditions.

The purpose of the research was to test whether agent-based modeling could successfully depict traders using the Black-Scholes formula in an options market subject to jump diffusion, and exhibit market behaviour useful for analysis and for helping to understand the complex interactions of the options market. An analysis of the current literature reveals that one research group has created an agent-based model of the Black-Scholes Formula but did not incorporate jump diffusion or use the NetLogo platform [8][10][12].

For the research presented in this paper, it was hypothesized that increasing jump diffusion would result in measurably "toxic markets" that impair traders' ability to successfully use the Black-Scholes Formula. The kurtosis was also predicted to rise when jump diffusion was present. It was hypothesized that kurtosis of an underlying asset price could signal if an options market was "toxic" by accurately and consistently detecting whether jump diffusion was in effect. An analysis of the current literature reveals that these hypotheses are unique and represent new advancements in the field. Since the model yields abstract results from a simulated market, findings must be quantified to benefit individuals who use option-contract-related methods. For this reason, the results of market runs with varying jump diffusion and volatility values were examined to find which combinations of variables can create unpredictable markets. To find the results, jump frequency, jump size, and jump variance were adjusted, along with the underlying asset price volatility, and, over several aggregated runs, the behavior of the market was observed.

After running the simulation, the results confirmed the hypothesis. Jump diffusion made the market more hazardous, as exemplified by analyzing several variables. In addition, the kurtosis proved to be a reliable indicator of whether jump diffusion was present. Overall, this paper shows that agent based tools such as NetLogo are useful for modeling and understanding both the intensive mathematics and the complex trading behavior in the options contract market. The mathematical modeling and simulation tool presented here provides an effective platform for analyzing such markets. From such analysis, one can determine the risks and strategies associated with differing market conditions.

The remainder of this paper is organized as follows. Chapter 2 provides a description of the simulation mathematics. The design of the custom simulation is explained in Chapter 3. Results from the simulation are presented in Chapter 4, along with sample data and an error analysis. Finally, Chapter 5 covers conclusions including possible directions for future research.

2. Equations for Black-Scholes Option Pricing with Jump Diffusion

2.1 Geometric Brownian Motion

Geometric Brownian motion is a mathematical calculation for a random walk [4]. A two-dimensional line graph of a random walk with geometric Brownian motion exhibits behavior similar to a stock price, which is why the Black-Scholes Formula is based on geometric Brownian motion. All the effects that perturb an asset price or other measured variable in nature are aggregated in to a single volatility value sigma.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW^p(t)$$

S = price of asset (dollars)
t = time (years)
μ = drift rate (unitless)

σ = volatility (unitless)
dW^p= stochastic differential equation for stock prices

2.2 Geometric Brownian Motion with Jumps

Geometric Brownian motion with jumps features an additional term that determines the size and frequency of an additional jump process beyond the random walk. A Poisson random distribution multiplied by J – 1 allows for the chance of a jump in the market [16].

$$dSt = \mu S(t)dt + \sigma S(t)dW_t + (J-1)S(t)dN(t)$$

S = price of asset (dollars)
t = time (years)
μ = drift rate (unitless)
σ = volatility (unitless)
dW^p= stochastic differential equation for stock prices
J = random process based on a normal distribution determining the jump size
N(t) = Poisson random process with intensity λ determining the jump frequency

2.3 Black-Scholes Option Pricing Equations

The Black-Scholes Equation mostly uses values that are known to all traders. However, the volatility is estimated from observance of the stock market behavior, so the actual volatility is hidden from the simulated traders. Black-Scholes gives what the price of the option should be as a result of the inputted variables. This result can be compared to the market price of the option to determine whether one should buy or sell. Derivation and proofs of the Black-Scholes Equation are available in several references [4][15][16] and are beyond the scope of this paper. For a call option, the trader has the option, but not the obligation, to buy the underlying stock at the strike price on the expiration date. For a put option, the trader has the option, but not the obligation, to sell the underlying stock at the strike price on the expiration date.

Call Option Equation for Black-Scholes

$$C(S,t) = N(d1)S - N(d2)Ke^{-r(T-t)}$$

Put Option Equation for Black-Scholes

$$P(S,t) = N(-d2)Ke^{-r(T-t)} - N(-d1)S$$

$$d1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d2 = d1 - \sigma\sqrt{T - t}$$

C( ) = call option price
P( ) = put option price
S = spot price (the price of the underlying stock)
K = strike price (the agreed upon price of the stock at maturity)
r = risk free rate of interest
σ = stock volatility
N( ) = normal standard distribution function
T - t = time until maturity

2.4 Jump Diffusion Option Pricing Equation

With jump diffusion present, the values σ and r in the Black-Scholes equation are replaced with σ_k and r_k, as calculated with the two equations below the main jump diffusion equation, which is a summation. As the value of k increases, the probability of k jumps happening decreases. In this way, the amount of money added to the new jump diffusion price converges to zero as k number of jumps happening becomes increasingly unlikely. The result of the jump diffusion equation gives how much to add to the value of the purely Black-Scholes price due to random jumps [16]. The presence of jump diffusion largely poses a threat because its associated variables cannot be calculated by individual traders, in contrast to the basic Black-Scholes variable sigma which can be estimated reliably by traders.

$$P_{JD} = \sum_{k=0}^{\infty} \frac{\exp(-m\lambda T)\,(m\lambda T)^k}{k!} P_{BS}(S, K, \sigma_k, r_k, T)$$

$$\sigma_k = \sqrt{\sigma^2 + kv^2/T}$$

$$r_k = r - \lambda(m - 1) + k\,log(m)/T$$

P_{JD} = price with jump diffusion
P_{BS} = price with Black-Scholes
v = jump volatility
k = number of jumps
λ = average number of jumps for each T-t
m = average jump size
T = time until expiration
S = spot price
K = strike price
r = original risk free rate
σ = original volatility

2.5 Kurtosis Equation

Kurtosis is equivalent to the calculation of volatility of volatility, or how much the underlying sigma value fluctuates. Essentially, kurtosis shows how much a distribution measures up to a normal distribution based on the peakedness or flatness of the distribution. A kurtosis departing from zero (positively or negatively) indicates that the option market sigma value is potentially unpredictably volatile, and the price behavior is departing from the constant-sigma geometric Brownian motion model upon which the Black Scholes formula relies. In this project, kurtosis was discovered to be an excellent detector of the presence of jump diffusion.

$$Kurtosis = F1 \sum \left(\frac{x_i - \bar{x}}{s}\right)^4 + F2$$

$$F1 = \frac{n(n+1)}{(n-1)(n-2)(n-3)}$$

$$F2 = -\frac{3(n-1)^2}{(n-2)(n-3)}$$

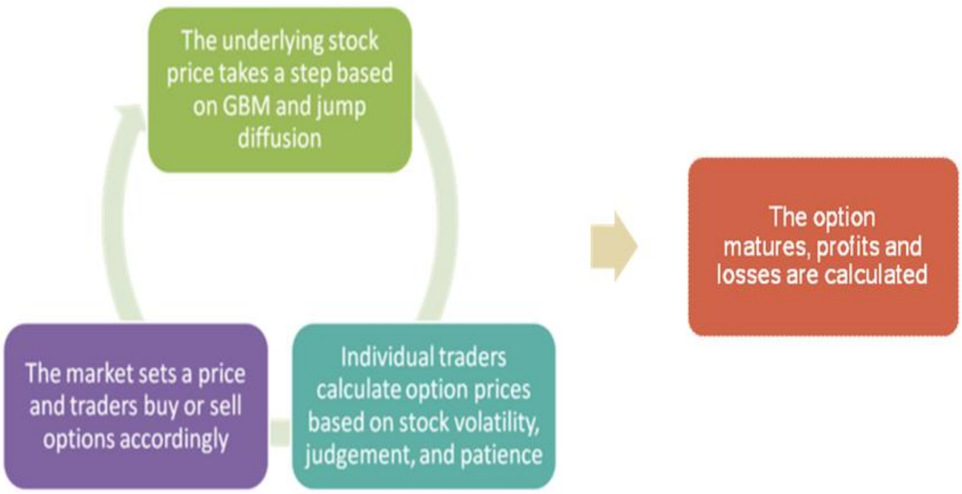
x_i = i^{th} value
$\bar{x}$ = sample mean
n = number of samples
s = sample standard deviation

3. Simulation Design

The model was programmed for this research using the agent-based modeling software NetLogo [21]. The NetLogo program creates multiple agents who interact within a simulated model. So-called agent based programs can be used for a wide variety of purposes, from simulating the spread of a disease to simulating a traffic grid. The Black-Scholes options market model created in NetLogo relies largely on the mathematical equations for geometric Brownian motion and the Black-Scholes formula, as well as the behavioral rules for simulated options traders. In this particular paper, an options market is defined as a group of individuals trading European stock options, which are stock options that can only be “cashed in” at the expiration date and not before.

Based on geometric Brownian motion, the Black-Scholes formula calculates the current value of a stock option using the variables of time until expiration, spot price, strike price, risk free rate of interest, and volatility. For the model, simulated options traders are created, each observing the underlying stock price, adjusting their calculations of the stock’s volatility using their individual “judgement” and “patience” values [10][12]. Judgement refers to how far the trader incorporates personal judgement and deviates from the Black-Scholes formula. Patience refers to how quickly the trader enters the market and how much of the past market the trader looks at in calculations, especially in considering the key unknown value in the formula: underlying stock volatility. Figure 1 gives an overview of the simulation activity, described in more detail below.

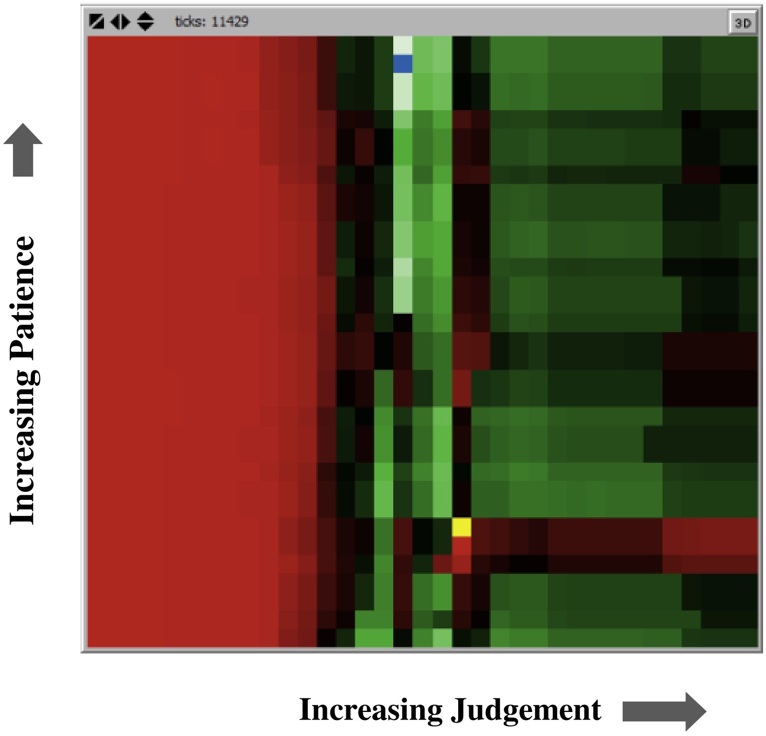
Figure 1: The cycle of the program, ending with the calculation of profits and losses



In this agent-based model of an options market, a plot was created mapping different traders on a grid, as shown in Figure 2 below. Each small square represents a trader. The x and y values of the traders on the plot indicate their judgement and patience, respectively. Each trader calculates an individual desired price based on the Black-Scholes Formula and each individual trader's own best estimate of volatility. Thus, the traders deviate according to their individual judgement and patience, allowing one to study how different personal variables affect success in the market as the traders seek more profitable operating points.

The traders are color coded depending on their success. Red signifies loss, while green signifies success. Lighter colors are on the extremes of success and loss. Conversely, the darker the color, the closer the trader is to zero profit or zero loss. The blue square indicates the most successful trader, and the yellow square indicates the least successful trader. Additionally, the user can adjust values such as the starting price or the actual volatility that is hidden from traders. Using the "Monte Carlo" method, markets are run quickly in succession, and the results of each market are averaged into the aggregate results of all the market runs.

Figure 2 : Simulator Plot of Judgement versus Patience



Traders enter the market when their patience in days has been reached. While the market runs, the spot price is updated with geometric Brownian motion plus jump diffusion. The traders update their minimum and maximum volatility if they see a new lowest or highest volatility, which in turn determines their buy and sell price. The market price of the option is found by taking the maximum buy price and minimum sell price of the traders and averaging the two. The traders look at the current options prices, and then they decide whether they want to buy or sell. At this point, the traders calculate with Black-Scholes and their estimated sigma what their buy and sell prices are, incorporating judgement. Their money and number of options contracts are both updated accordingly.

The program then finds the best and worst values for judgement and patience among the traders and indicates those two values on the graph (blue for the most successful trader, yellow for the least successful trader). The current true price is also calculated for the user to view, using the actual sigma value hidden to traders. The program then finds the pricing bias (how far off the market price is from the price calculated with the actual volatility), and includes that in the mean pricing bias over time. Finally, the program calculates the current profit of the traders. Throughout the processes of running the market, the program updates the graph of stock price, the graph of the options market, the sigma (volatility) plot with minimum and maximum sigma, and the rate of activity in terms of buying and selling.

Jump diffusion techniques add an extra dimension to the program [18]. Adjustable variables direct the magnitude, frequency, and variance in the jumps that occur. Lambda increases the frequency of the jumps, and with standard time settings, lambda is the average number of jumps per year. The value ‘m’ gives the average size of the jump, with values below one giving negative jumps on average and a value of one giving equal probability of positive or negative jumps. The value ‘v’ is the jump volatility, which adjusts the variation in the size of the jumps. With the jump diffusion variables, one can analyze an options market with random jumps or crashes to examine the impact on the Black-Scholes formula’s success.

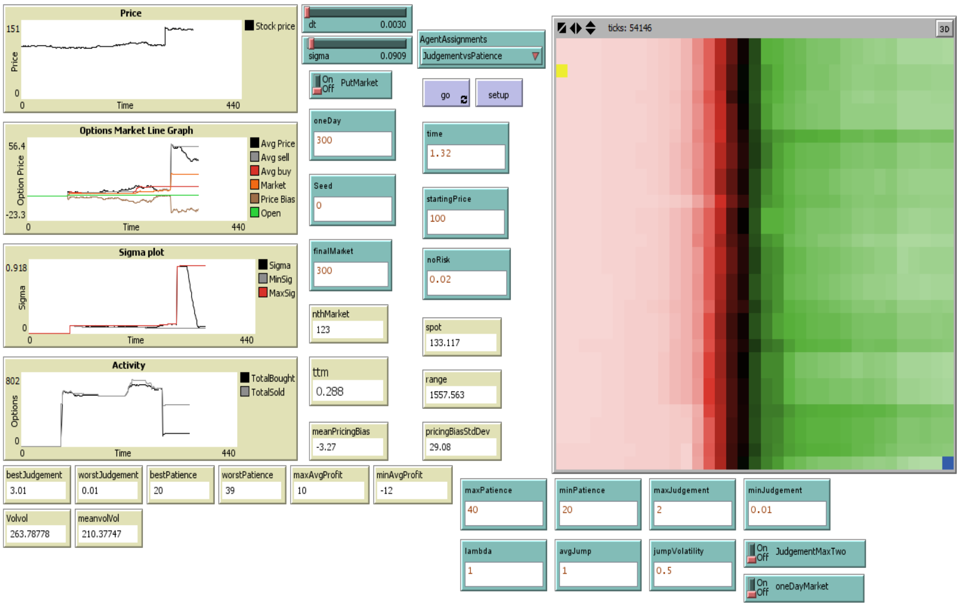
Kurtosis calculates the value of volatility of volatility [19]. Volatility of volatility is used to determine how "toxic" a market is. A toxic market is defined as one that is unstable or difficult to predict, making it undesirable to participate in such a market. Under high volatility of volatility, the underlying volatility becomes more elusive because it varies more. When the underlying stock volatility is harder to pin down, Black-Scholes becomes less effective. Therefore, markets with high volatility of volatility are toxic markets—these markets are best avoided because traders cannot use robust analytical techniques to combat random chance.

4. Results

4.1 Simulation example

Figure 3 below shows the user interface of the model, captured after 123 market runs have been aggregated. The main display discussed earlier is located to the right. Recall that judgement increases from left to right, and patience increases as one moves upward. The red on the graph indicates unsuccessful traders, while the green represents successful traders. In addition, the traders with higher judgement values have mostly done better in this market, as the graph shifts from red to green as one moves from left to right. The asset price graph in the upper left corner also displays a jump, and the other graphs show the jump’s effect on the option prices and the estimate of sigma.

Figure 3 : Screenshot of options market simulator



4.2 Simulation Analysis

To focus on the effects of jump diffusion, the following parameters were held constant in the simulation runs presented in this chapter: option time to expiration = 1 year, simulation time step = 1 day, risk free rate of interest (μ, drift) = 0.02, minimum and maximum judgement and patience values, initial spot price = 100, strike price = 100, and the number of market runs = 500. The variables and constants are summarized in the table below.

Table 1 : Variables and constants used to analyze the effects of jump diffusion

Independent Variables	Constants	Dependent Variables
• Jump Volatility (variance in jumps) • Lambda (average number of jumps per year) • Stock Volatility (variance in the underlying stock price)	• Time until Expiration and the timestep • Average Jump Size • Mu (drift and risk free rate) • Maximum and minimum possible judgement and patience values • Initial Spot Price • Initial Strike Price	• Mean Kurtosis • Max-Min Average Profit • Mean Pricing Bias • Pricing Bias Standard Deviation • Best judgement

Figure 4 : Mean Kurtosis

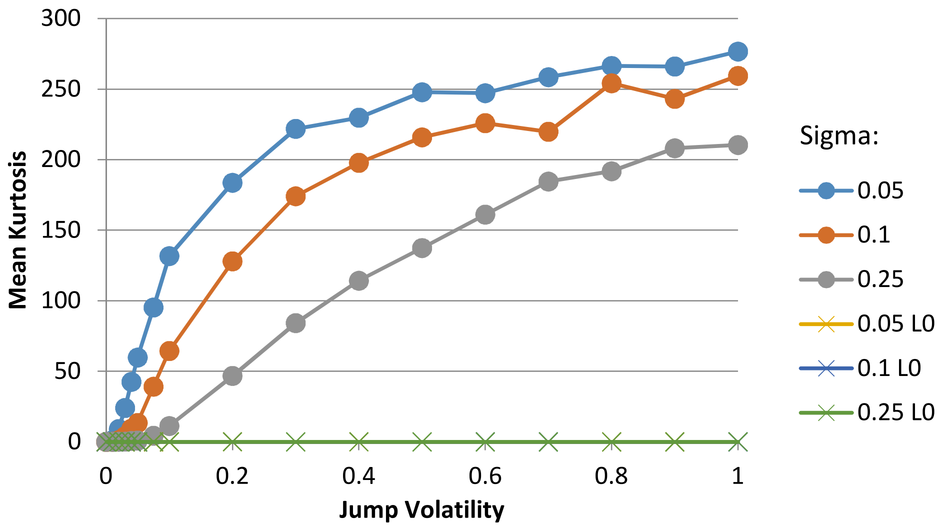
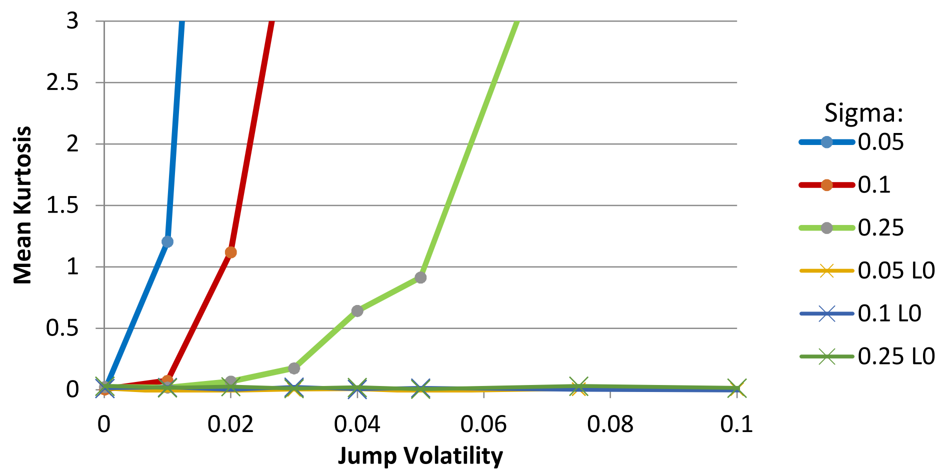


Figure 5 : Mean Kurtosis, Detail at Low Jump Volatility



The graphs in Figures 4 and 5 portray the volatility of volatility, or kurtosis, measured for a call options market averaged over 500 market runs for various sigma and jump volatility values. Lambda was alternated between lambda = 0 (no jumps, for the purple, light blue, and gold lines, which are virtually collinear) and lambda = 1 (one jump per year on average, for the blue, red, and green curves). As shown, even with just one jump per year, mean kurtosis spikes at low jump volatility values, thereby making mean kurtosis an effective measure of the presence of jump diffusion. As predicted by theory, this graph illustrates that underlying asset volatility sigma has a negligible effect on mean kurtosis, especially compared to jump diffusion. Therefore, mean kurtosis gives a clear indication of whether jump diffusion is present.

Figure 6 : Max-Min Average Profit, Lambda = 1

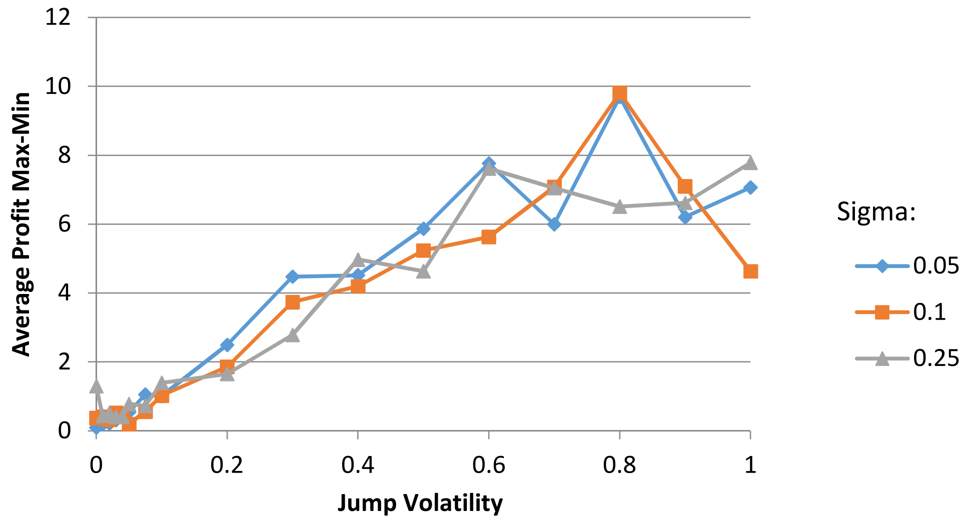
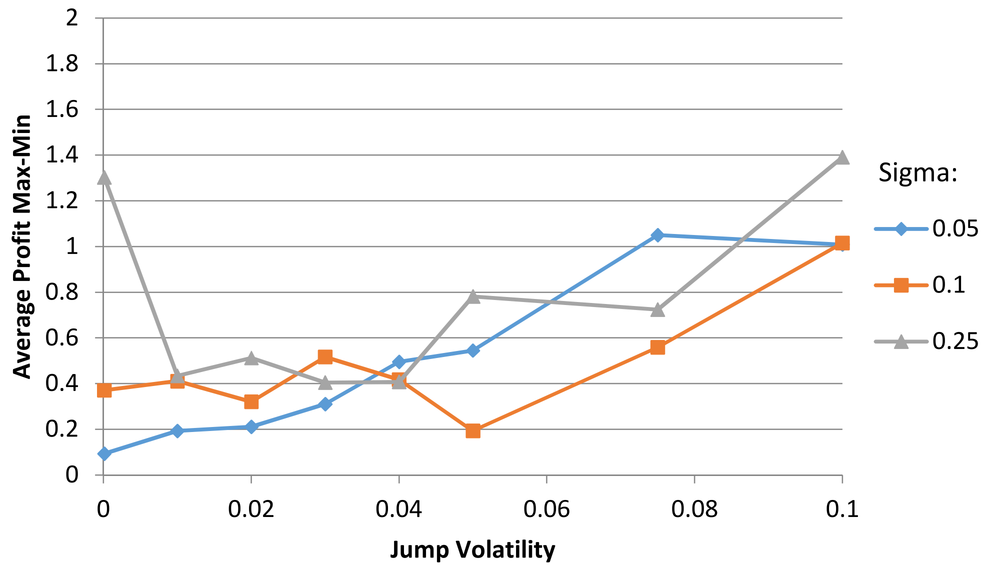


Figure 7 : Max-Min Average Profit, Lambda=1, Detail at Low Jump Volatility



Figures 6 and 7 show the difference between maximum profit (most successful trader) and minimum profit (least successful trader), in a given market run, averaged over 500 market runs for a lambda of 1 (one jump per year on average). As the graphs show, this max-min profit value increases with jump volatility, as the effects of jump diffusion create a wider disparity between the most and least successful traders. As a result, jump diffusion increases how much the option could change from its initial price. Therefore, those who wish to avoid losing money from excessive market swings would do best to stay out of such a market.

Figure 8 : Mean Pricing Bias, Lambda = 1

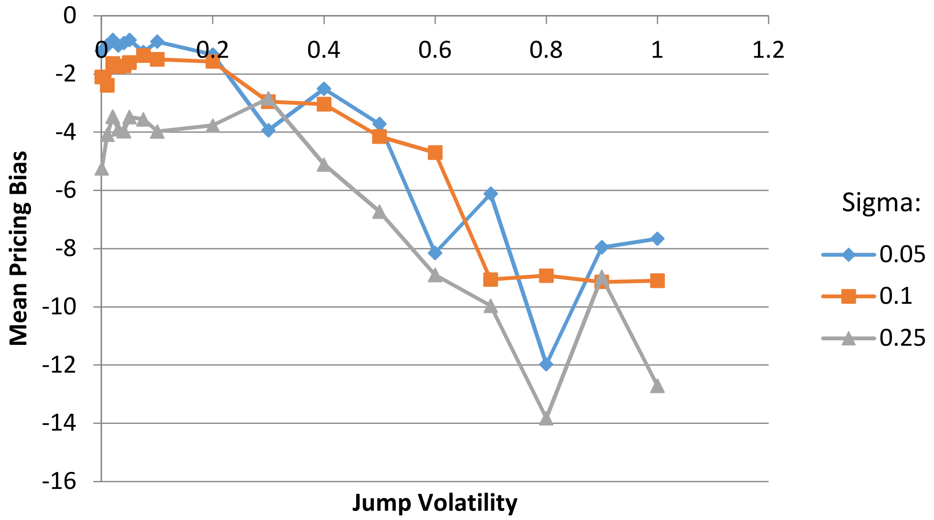
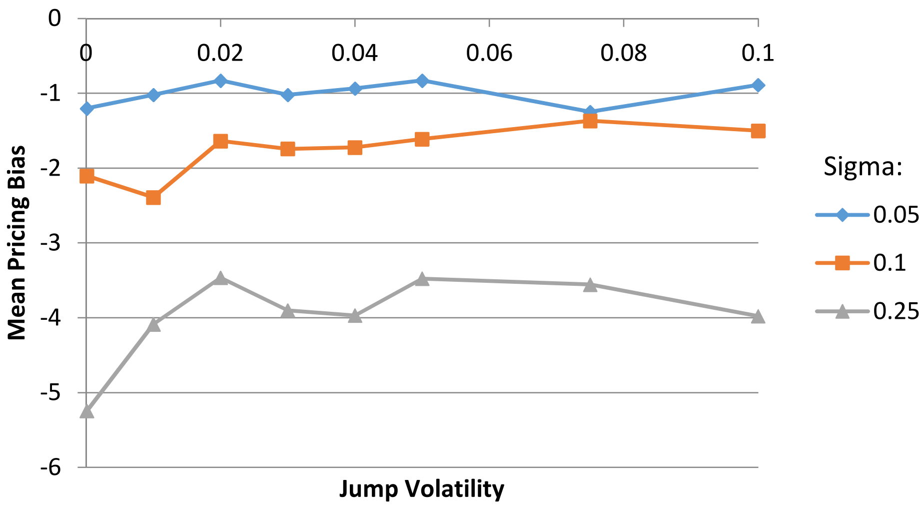


Figure 9 : Mean Pricing Bias, Call Options, Detail at Low Jump Volatility



Figures 8 and 9 show the mean pricing bias increasingin magnitude with higher jump volatility, as the traders drive the market price far from the pure Black-Scholes theoretical price. The pricing bias, or difference between market price and theoretical Black-Scholes price, was averaged over 500 market runs. Moving away from the basic Black-Scholes calculation implies that another force besides underlying asset volatility (sigma) is impacting the traders. In this case, that force must be jump diffusion destabilizing the market.

Figure 10 : Best Judgement, Lambda = 1

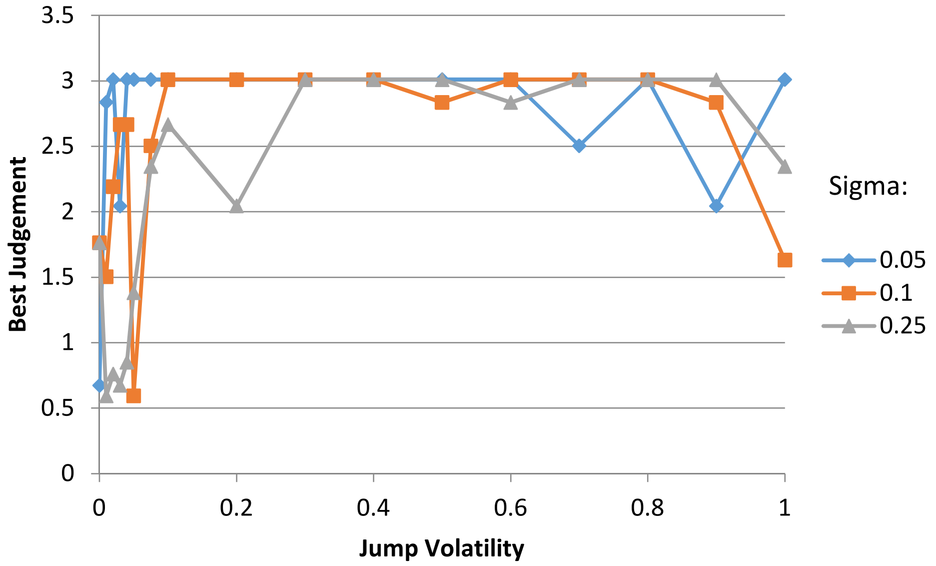
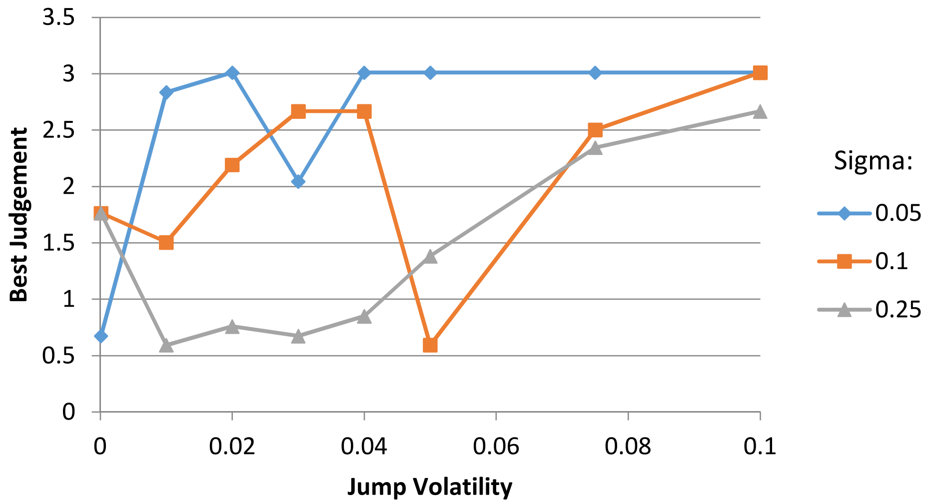


Figure 11 : Best Judgement, Lambda=1, Detail at Low Jump Volatility



Figures 10 and 11 show the best judgement value, which is the judgement value applied by the trader that ends up with the highest profit at the end of a market run, averaged over 500 market runs. The best judgement value quickly hits the ceiling of 3.01 as jump volatility increases. As early as jump volatility=0.04, best judgement is at the maximum allowed by the pricing model. This result indicates an attempt to increase the price due to the additional volatility. Best judgement shows another reason why the presence of jump diffusion can be considered toxic.

Figure 12 : Pricing Bias Standard Deviation, Lambda=1

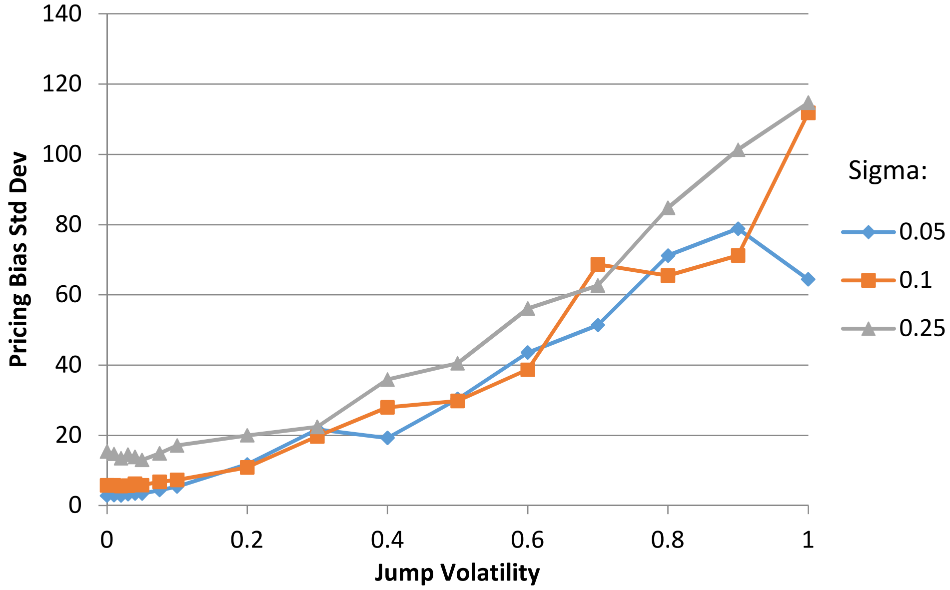
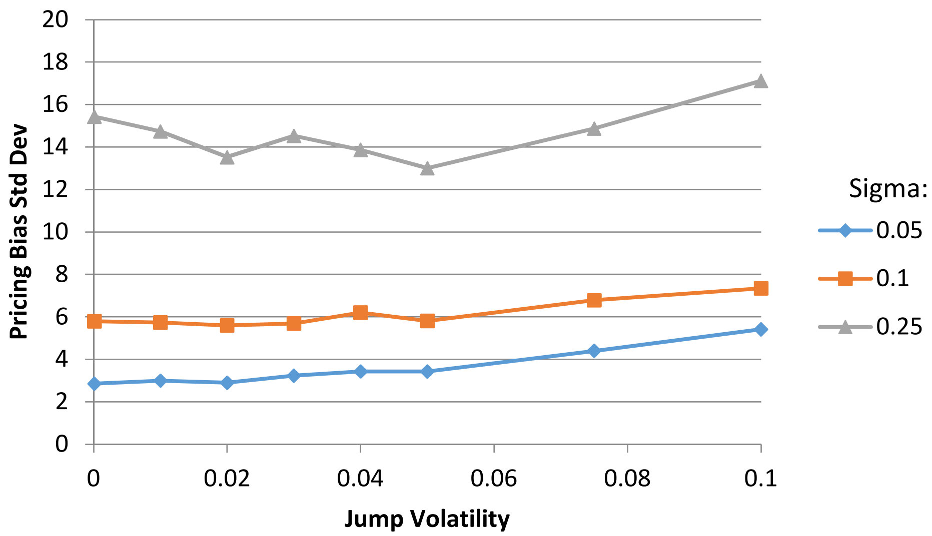
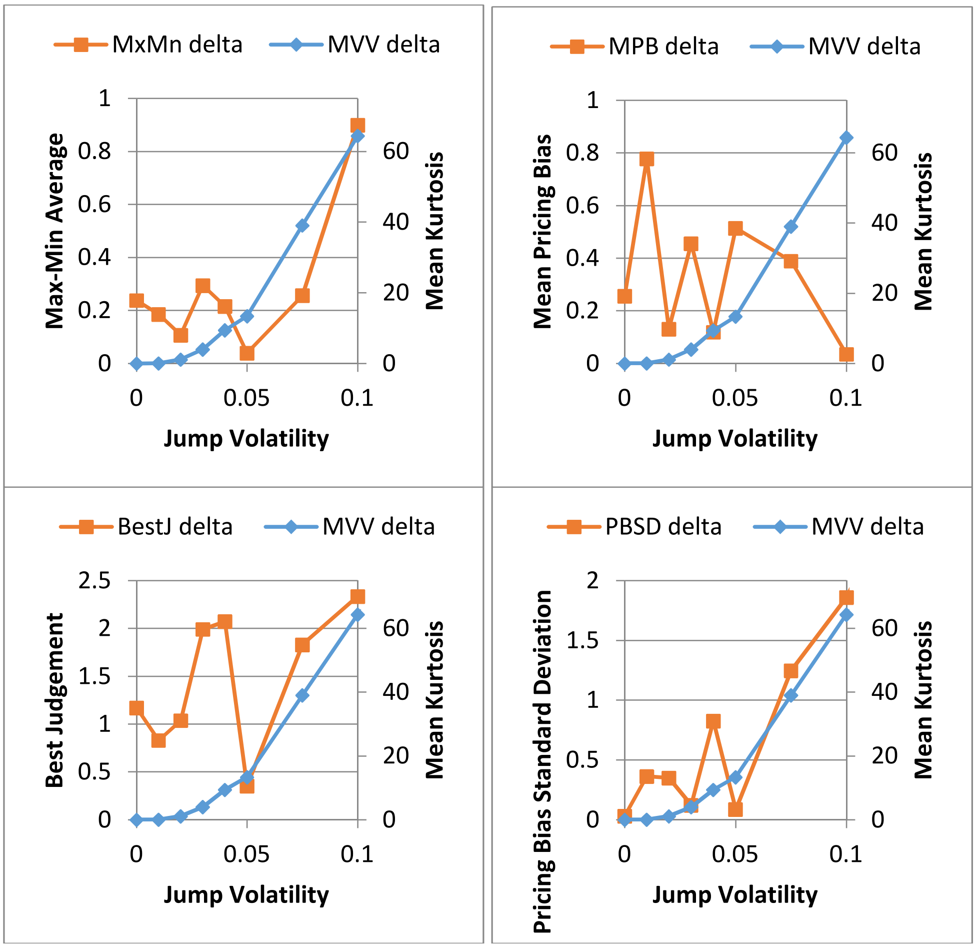


Figure 13 : Pricing Bias Std Dev, Lambda=1, Detail at Low Jump Volatility



Figures 12 and 13 show the standard deviation of pricing bias also increases with jump volatility, which means the traders spread the market price farther apart as they try to use Black-Scholes in the presence of jump diffusion. Entering a market with high pricing bias standard deviation means traders find it difficult to pinpoint a successful price.

Figure 14 : Kurtosis Compared to Other Jump Diffusion Metrics at Low Jump Volatility



For the four graphs in Figure 14, the market type was a call options market and sigma was 0.1. The "MVV delta" line subtracted mean kurtosis when lambda was 1 from mean kurtosis when lambda was 0 under these conditions. Each “delta” line likewise subtracted its respective value when lambda was 1 from the value when lambda was 0 under these conditions. Distance from the x-axis represents how strongly the plotted variable is affected by lambda's presence under a given jump volatility value. All of these variables increase with jump volatility, as kurtosis does. However, kurtosis increases more smoothly and decisively than any of the other variables. Although all of the values can detect the presence of lambda to some degree, mean kurtosis is the clearest and strongest indicator. "MVV delta" curves upward in a smooth plot, but the other plots are irregular and fluctuate erratically. In addition, unlike kurtosis, none of the other variables shown here besides kurtosis can be analyzed by an individual trader.

The put market data displays results similar to the call option data analyzed above. Whether the market involves call or put options, jump diffusion ultimately has the same effect of unbalancing the market. Since the put market data is largely similar to the call option data, it was not graphed in this paper.

4.3 Error Analysis

For a call options market under a lambda of 1 and a sigma of 0.5, the R^2 value for a binomial approximation of mean kurtosis was 0.984. This means that the plot of mean kurtosis was very consistent and accounted for 98.4% of the variation in the plot. For comparison, in a call options market under a lambda of 1 and a sigma of 0.5, plotting mean pricing bias gave an R^2 value of 0.917. Although mean pricing bias clearly increases with jump diffusion, kurtosis yields a more consistent association than mean pricing bias. Kurtosis has a superior association compared to other metrics such as pricing bias standard deviation and best judgement as well. Additionally, regarding number of market runs used to compute results, the intent of the study was to determine trends rather than boundaries. Therefore, 500 market runs was deemed enough to provide a clear indication of market behavior. Furthermore, preliminary testing with 500 market runs yielded expected patterns under a Black-Scholes system.

4.4 Simulation Data Examples

Table 2 : Call Market Data, Averaged from 500 Simulated Market Runs

Input			Output				
Lambda	Jump Volatility	Sigma	Mean Kurtosis	Max-Min Avg Profit	Mean Pricing Bias	Pricing Bias Std Dev	Best Judge-ment
1	0.00	0.05	-0.003	0.093	-1.203	2.851	0.674
1	0.10	0.05	131.711	1.009	-0.889	5.416	3.010
1	0.20	0.05	183.632	2.493	-1.334	11.791	3.010
1	0.40	0.05	229.790	4.516	-2.509	19.266	3.010
1	0.60	0.05	247.153	7.766	-8.147	43.586	3.010
1	0.80	0.05	266.455	9.693	-11.972	71.175	3.010
1	1.00	0.05	276.654	7.062	-7.657	64.433	3.010
1	0.00	0.10	0.003	0.370	-2.104	5.795	1.764
1	0.10	0.10	64.431	1.015	-1.499	7.348	3.010
1	0.20	0.10	127.966	1.855	-1.569	10.904	3.010
1	0.40	0.10	197.802	4.194	-3.042	27.954	3.010
1	0.60	0.10	225.937	5.625	-4.701	38.681	3.010
1	0.80	0.10	254.145	9.806	-8.930	65.498	3.010
1	1.00	0.10	259.436	4.630	-9.099	111.802	1.632
1	0.00	0.25	0.024	1.302	-5.244	15.426	1.764
1	0.10	0.25	11.249	1.391	-3.977	17.123	2.667
1	0.20	0.25	46.756	1.651	-3.769	20.010	2.045
1	0.40	0.25	114.185	4.972	-5.112	35.894	3.010
1	0.60	0.25	161.081	7.615	-8.905	56.111	2.836
1	0.80	0.25	191.812	6.510	-13.826	84.826	3.010
1	1.00	0.25	210.432	7.787	-12.707	114.777	2.346
0	0.00	0.05	0.010	0.105	-1.122	2.906	1.048
0	0.10	0.05	0.005	0.116	-1.067	3.076	0.674
0	0.20	0.05	0.015	0.128	-1.095	2.766	0.851
0	0.40	0.05	0.016	0.116	-0.996	2.894	0.594
0	0.60	0.05	-0.001	0.183	-1.149	3.114	1.505
0	0.80	0.05	0.012	0.178	-1.208	3.015	1.154
0	1.00	0.05	-0.018	0.091	-1.143	2.909	0.594
0	0.00	0.10	0.011	0.132	-1.848	5.825	0.594
0	0.10	0.10	-0.006	0.116	-1.534	5.486	0.674
0	0.20	0.10	-0.004	0.194	-1.714	5.204	0.760
0	0.40	0.10	-0.012	0.229	-1.597	5.338	1.048
0	0.60	0.10	0.016	0.260	-1.672	5.571	1.632
0	0.80	0.10	-0.006	0.200	-1.845	5.141	1.048
0	1.00	0.10	0.010	0.289	-1.988	5.446	1.505
0	0.00	0.25	0.030	0.382	-3.760	15.044	0.674
0	0.10	0.25	0.013	0.432	-3.618	13.215	1.902
0	0.20	0.25	0.028	0.424	-2.923	13.339	0.594
0	0.40	0.25	0.016	0.460	-3.814	14.965	0.674
0	0.60	0.25	0.004	0.740	-4.520	15.249	1.154
0	0.80	0.25	0.004	0.632	-3.958	14.492	0.760
0	1.00	0.25	0.020	0.421	-3.259	13.684	0.947

Table 3 : Put Market Data, Averaged from 500 Simulated Market Runs

Input			Output				
Lambda	Jump Volatility	Sigma	Mean Kurtosis	Max-Min Avg Profit	Mean Pricing Bias	Pricing Bias Std Dev	Best Judge-ment
1	0.00	0.05	-0.011	0.069	-0.364	1.685	1.764
1	0.10	0.05	129.689	1.057	-0.265	3.606	3.010
1	0.20	0.05	188.622	1.934	0.092	5.862	3.010
1	0.40	0.05	234.062	4.714	-0.813	11.415	3.010
1	0.60	0.05	251.238	6.795	-3.706	15.006	3.010
1	0.80	0.05	256.487	8.403	-7.639	17.362	3.010
1	1.00	0.05	268.562	8.535	-8.476	18.223	3.010
1	0.00	0.10	-0.011	0.150	-0.884	4.064	0.519
1	0.10	0.10	59.814	0.957	-0.957	5.524	3.010
1	0.20	0.10	129.481	2.150	-0.203	7.136	3.010
1	0.40	0.10	192.369	4.481	-2.084	11.506	3.010
1	0.60	0.10	217.752	7.376	-4.206	15.266	3.010
1	0.80	0.10	242.483	9.062	-6.946	18.152	3.010
1	1.00	0.10	248.351	9.285	-8.239	18.152	3.010
1	0.00	0.25	0.019	0.326	-3.434	9.643	0.947
1	0.10	0.25	9.709	1.016	-3.118	9.814	1.383
1	0.20	0.25	46.167	1.743	-2.448	10.408	2.193
1	0.40	0.25	116.153	3.551	-2.753	14.218	3.010
1	0.60	0.25	160.431	5.740	-3.881	15.519	3.010
1	0.80	0.25	188.525	7.722	-6.968	18.234	3.010
1	1.00	0.25	225.169	8.956	-10.795	20.049	3.010
0	0.00	0.05	0.000	0.065	-0.437	1.815	1.505
0	0.10	0.05	-0.012	0.037	-0.449	1.726	0.674
0	0.20	0.05	0.005	0.051	-0.375	1.608	3.010
0	0.40	0.05	0.006	0.044	-0.308	1.599	0.519
0	0.60	0.05	-0.003	0.036	-0.283	1.631	0.519
0	0.80	0.05	-0.007	0.081	-0.368	1.665	1.764
0	1.00	0.05	-0.014	0.079	-0.535	1.731	2.667
0	0.00	0.10	0.004	0.144	-1.249	3.781	1.266
0	0.10	0.10	0.001	0.092	-0.898	3.674	0.674
0	0.20	0.10	-0.007	0.082	-0.776	3.552	0.594
0	0.40	0.10	-0.002	0.265	-1.246	3.984	1.632
0	0.60	0.10	-0.014	0.163	-1.084	3.668	1.505
0	0.80	0.10	0.012	0.153	-1.131	3.841	2.045
0	1.00	0.10	0.014	0.120	-1.061	3.796	1.266
0	0.00	0.25	0.006	0.277	-2.609	8.863	0.851
0	0.10	0.25	0.005	0.273	-2.931	9.036	0.851
0	0.20	0.25	0.016	0.440	-3.039	9.208	1.383
0	0.40	0.25	0.014	0.302	-3.125	9.488	0.851
0	0.60	0.25	0.018	0.389	-2.425	8.385	0.674
0	0.80	0.25	0.007	0.254	-2.346	8.874	0.674
0	1.00	0.25	0.016	0.447	-2.934	9.346	1.154

5. Conclusions

5.1 Discussion

Comparing sigma, jump volatility, pricing bias, average profit variables, and best judgement, a few conclusions can be drawn. The difference between maximum and minimum average profit increases with jump volatility, which means that trader profits become more spread out with jump diffusion. The fact that the standard deviation of mean pricing bias increases with jump volatility reinforces this result. The absolute value of mean pricing bias also rises with jump volatility, signifying how the market moves away from the basic Black-Scholes calculation when jump diffusion is present. Best judgement likewise increases greatly, as the best price moves away from the basic Black-Scholes calculation. As noted in earlier work [22], higher jump volatility increased the best judgement value. In other words, traders who set the price of options as higher than their vanilla Black-Scholes result unwittingly incorporated the concept that higher jump volatility should make the price of the options increase, which made traders with higher judgement more successful. All of these results, taken from values unknowable to traders or unknown until the end of a market run (option contract period), signify how jump diffusion makes an options market much more unpredictable, causes trading difficulties, and thus is considered "toxic."

A substantial result of this work is the discovery that kurtosis, or volatility of volatility, among all the measures of market behavior, is a precise detector of the presence of jump diffusion. When lambda was 0, and therefore when there were no jumps in the market, the mean kurtosis stayed close to 0 as well, never going over 0.05 or under -0.05. When lambda was 1, the mean kurtosis was higher than 0.05 with a jump volatility of 0.02 or greater. Under a sigma value of 0.05 and a jump volatility value of 0.1, mean kurtosis climbed as high as 131.71, much larger than the value of 0.0055 that accompanied a lambda of 0. Furthermore, mean kurtosis was more sensitive than mean pricing bias or other market measurements at detecting jump diffusion. From these results, it is clear that calculating mean kurtosis consistently detects whether a market contains jumps if there is even a small amount of jump volatility. Therefore, mean kurtosis, which can be calculated from the underlying asset price like sigma, can be used as a clear and consistent indicator of jump diffusion. Such an indicator can be used to avoid the toxic market effects of jump diffusion.

Thus, the results supported the hypotheses. As these results show, agent-based modeling using NetLogo can successfully depict traders using the Black-Scholes formula in an options market subject to jump diffusion, and exhibit market behavior useful for analysis and for helping to understand the complex interactions of the options market. Increasing jump diffusion results

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in measurably "toxic markets" that impair traders' ability to successfully use the Black-Scholes formula. The kurtosis of an underlying asset price can signal if an options market is toxic by detecting whether jump diffusion is in effect.

5.2 Future Directions

Additional useful information could be found by measuring the analyzed variables at different values of lambda. As lambda increases and the jumps become more regular, jump diffusion can "set a new sigma." Therefore, it would be useful to test the model at higher lambda values to determine whether kurtosis remains effective. In this case, the market could be analyzed to find if jump diffusion eventually replaces sigma to the extent that the market becomes safe. Real world options markets could be further explored to test the robustness of kurtosis and determine if other variables may give the kurtosis a "false alarm" in its measure of jump diffusion. Other possibilities include making use of the additional behavior rules available in the NetLogo platform to model more complex trader behaviors, or developing a mathematical description of the price similar to Black-Scholes but incorporating kurtosis.

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