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A Novel Technique for Studying the Shear Elastic Properties of Weak Solids

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A Novel Technique For Studying The Shear Elastic Properties of Weak Solids

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Abstract

We have developed a simple, inexpensive, and precise technique to measure the shear elastic modulus of weak solids using electromagnetic and optical tools. This technique can be easily adapted to measure the viscosity of a liquid also. A Helmholtz pair was used to produce a torque on a permanent magnet mounted on the smaller of two concentric cylinders, coupled by the material to be studied. The torque was controlled precisely and measured accurately in terms of the current flowing through the coils of the Helmholtz pair. An optical lever was employed to measure the angular displacement of the inner cylinder as a function of the applied shear stress. The instrument has been validated by making measurements on lemon jello, and agarose gels of varying concentrations. The technique has also been applied to the study of electric field induced “freezing” of electrorheological fluids, a subject of enormous contemporary interest.

Introduction

Traditionally, the shear elastic properties of solids have been measured using mechanical instrumentation. The material under study is typically grabbed and then twisted or stretched. The angular displacement, or change in length, is then measured by either mechanical or electronic methods. Results of such measurements are frequently presented as stress-strain ($\tau$-$\gamma$) graphs, where $\tau$ denotes the shear stress and $\gamma$ represents the shear
strain (Figure 1). A solid typically behaves linearly over a finite region of stresses and eventually moves into a non-linear regime. This non-linear behavior terminates when the material fails.

![Diagram showing linear and non-linear regions of stress-strain behavior](image)

**Figure 1**

The slope of the linear regime of a stress versus strain graph is then the elastic modulus of the material under study. We are interested in the static shear modulus $G$ defined as:

$$G = \lim_{\gamma \to 0} \frac{\Delta \tau}{\Delta \gamma}$$  \hspace{1cm} \text{(Equation 1)}

where $\dot{\gamma}$ is the time rate of change of strain and is called the strain rate.

Gels, foams, and electrorheological fluids are typical examples of a class of materials that have customarily been grouped under the label "complex fluids." The importance of these materials in such areas as automotive engineering, robotics, and chemical engineering warrants close investigation of their properties under different conditions. They are also of fundamental interest to materials scientists because their properties place them at an intermediate stage between solids and liquids. Shear elasticity of such weak solids is the
focus of this paper. This is an obvious characteristic to study because it probes the underlying structure of these weak solids at the microscopic level.

The study of shear elastic properties of weak solids requires the application of extremely small stresses and the measurement of minute strains. Commercially available instruments are designed to study engineering materials, like steel, and are not useful for investigating weak solids. These machines are inappropriate because they cannot apply and measure the small stresses needed to study these delicate materials. Thus a new technique needed to be developed in order to study the shear properties of weak solids.

**Approach**

We wanted to design an instrument that enabled us to apply and measure stresses that were much smaller than those supplied by a typical commercial instrument. The instrument also needed to be simple, easy to build and fairly inexpensive. A goal of this project was to develop a technique to measure the shear elastic response of weak solids that can be duplicated in an undergraduate laboratory.

Our first concern was the shearing geometry. A solid can be sheared by either twisting it, or by moving two parallel faces in opposite directions (Figure 2).

![Twisting Shear vs Lateral Shear](image)

**Figure 2**
We opted to twist. It was then decided that a concentric cylinder configuration would be used to shear the material. The shearing cell consists of a stationary outer cylinder and an inner cylinder attached to a precision ball-bearing mutually coupled by the material under study (Figure 3).

The Instrument

![Diagram of the experimental setup](image)

*Figure 3* The experimental set-up for measuring the shear elastic response of weak solids. The inset shows an enlarged view of the shearing cell.

We next needed to decide how to stress the solid. A magnet was mounted on the inner cylinder and the shearing cell was placed in the center of the coils of a Helmholtz pair. A Helmholtz pair was chosen specifically because of the unique characteristics of the magnetic field produced by such a configuration.
The magnetic field due to a Helmholtz Pair is:

\[ B_H = \frac{\mu_0 NI}{R \left( \frac{5}{4} \right)^{3/2}} \]  

(Equation 2)

where:
- \( B_H \) = magnetic field (Gauss)
- \( \mu_0 \) = permeability of free space
- \( I \) = current through the coils (amps)
- \( R \) = radius of one coil (meters)
- \( N \) = number of turns in one coil

The gradient of \( B_H \), \( \partial B_H / \partial x \) is zero, where \( x \) is the distance from the center of one of the coils. And with a Helmholtz pair, \( x \) is equal to half the radius, \( R \), of a coil. A gradient of zero, implies that the magnetic field of a Helmholtz pair is spatially uniform. Therefore, a magnet placed in the field will experience a pure torque without being affected by any lateral forces.

A computer controlled power supply allowed for precise control and accurate measurement of the current flowing through the coils of the Helmholtz pair, which in turn controls the torque on the magnet. Since a d.c. current can be easily controlled from milli-amperes to many amperes, it allowed for a broad range of torques.

Finally, we needed to decide how to measure the angular displacement of the solid. Our choices ranged from potentiometers and pointers to optical levers and interferometers. We opted for the optical lever. Although less precise than interferometric techniques, the optical lever allows us to track large displacements, unlike the very limited range of an interferometer. It also kept the instrument relatively simple and inexpensive.
Results

In order to validate the technique, we wanted to test the instrument with a weak solid that was close to the liquid/solid phase boundary. We chose lemon jello, which had the properties we were looking for. A typical data set for lemon jello is shown in Figure 4.

![Shear Elastic Response of Lemon Jello](image)

**Figure 4**

A well defined solid-like response was indicated, viz. for $\tau \neq 0$, $\dot{\gamma} = 0$, and $\gamma \neq 0$. In words, the jello supported a finite shear stress, $\tau$ without flowing (i.e. $\dot{\gamma} = 0$). Further, the stress-strain relationship is quite linear and a well defined shear modulus, $G$, can be extracted from the slope (Equation 1). This was far from obvious at the outset of our study. It is possible that measurements over a broader range of strains will reveal more complex behavior.

The uncertainty along the strain axis is indicative of the fact that a raw, unfocused laser beam was used in the optical lever. This can be improved by using a collimator to further focus the beam and by increasing the distance to the meter stick used in the measurements. An interferometer would give very high accuracy for strain measurements, but would limit the range of such a tool.
Agarose gel was used to further validate the technique. This material is used in biology as a growth medium for bacteria. It is particularly easy to use because it is a gel at room temperature and only needs to be warmed slightly to make it a liquid again. This therefore makes it easy to pour the material into the annular space where it is allowed to gel in place. A nice feature of agarose gel is that the elastic constant of the material varies with the concentration of agarose. Typical results of these measurements are shown in Figure 5.

![Shear Elastic Response of Agarose](image)

**Figure 5**

Note that as the concentration of agarose increases, the slope of the $\tau$-$\gamma$ graph increases. This implies that the shear modulus of agarose gel increases rapidly in this range of concentrations. This qualitatively agrees with the expectation that the cross-linking density increases at the microscopic level with an increase in concentration. A detailed study of the concentration dependence of the shear modulus of agarose gel is currently in progress. We consider this adequate validation of the technique for the present. We were able to apply and measure the small stresses needed to properly study the shear elastic response in weak solids, while maintaining the simplicity and affordability of the instrument.
An Interesting Application of the Technique

Figure 6 shows so far the most interesting application of this instrument.

![Graph](https://via.placeholder.com/150)

**Figure 6**

It shows the elastic response of a class of fluids called electrorheological fluids in the presence of an electric field. These are colloidal suspensions of particles of high dielectric constant neutrally buoyant in an oil of smaller dielectric constant. Their response to an electric field is frequently described as a very large increase in their viscosity. Recently, however, various groups have begun to describe them more in the language of solids. For example, there have been attempts to measure the yield stresses and shear moduli of these materials in the presence of an electric field. Frequently the results are not reproducible and the contemporary research community often debates if there is any yield stress at all. Indeed, there are claims that the material behaves essentially like a liquid and has no yield stress down to the smallest strain rates that have been measured. We believe that these claims represent more of an inability to apply very small stresses and very small strain rates. As shown in Figure 6, the suspension of corn starch in mineral oil, in an electric field of 6 kv/cm, shows unambiguously a linear, solid-like shear elastic response from which a shear modulus can be extracted. The dashed line at and above a current of
0.5 amperes corresponds to what one might call a yield stress. It would not have been possible to delineate this well-defined solid-like behavior, including a linear regime and recoverable strain, if one could not make measurements for very small stresses and strains.

In the coming months, we intend to explore the mechanical response of ER fluids extensively using this instrument for various applied electric fields, various concentrations of the suspension, and for different ER fluids.

**Conclusion**

We have designed and built a simple and inexpensive instrument that can apply small stresses and measure small strains in order to study the shear elastic response of weak solids. Through the use of electromagnetic and optical tools we have achieved sufficient precision to investigate contemporary research problems, such as electrorheological "fluids."

Plans to improve the sensitivity and accuracy of our instrument are currently being sketched out. The raw, unfocused laser beam will be focused further by a collimator in the near future. This will also enable us to obtain more precise measurements of the angular displacement of the inner cylinder. Other improvements will include the use of better precision bearings, or perhaps the use of magnetically levitated bearings.

We will also apply this technique to measure the viscosities of different liquids. The instrument will incorporate two Helmholtz pairs driven by sinusoidal a.c. currents that are in quadrature, viz. 90 degrees out of phase. This will produce a rotating magnetic field at the inner cylinder and maintain a constant shear strain.

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References

