



5-4-1964

Deflection of an Electron Beam by Photons

Danning W. Bloom '64
Illinois Wesleyan University

Follow this and additional works at: https://digitalcommons.iwu.edu/physics_honproj



Part of the [Physics Commons](#)

Recommended Citation

Bloom '64, Danning W., "Deflection of an Electron Beam by Photons" (1964). *Honors Projects*. 13.

https://digitalcommons.iwu.edu/physics_honproj/13

This Article is protected by copyright and/or related rights. It has been brought to you by Digital Commons @ IWU with permission from the rights-holder(s). You are free to use this material in any way that is permitted by the copyright and related rights legislation that applies to your use. For other uses you need to obtain permission from the rights-holder(s) directly, unless additional rights are indicated by a Creative Commons license in the record and/ or on the work itself. This material has been accepted for inclusion by faculty at Illinois Wesleyan University. For more information, please contact digitalcommons@iwu.edu.

©Copyright is owned by the author of this document.

ILLINOIS WESLEYAN UNIVERSITY

DEFLECTION OF AN ELECTRON BEAM BY PHOTONS

SUBMITTED FOR PARTIAL
FULLFILLMENT OF THE
SENIOR HONORS PROGRAM

ARCHIVES

535.7

B655d

DEPARTMENT OF PHYSICS

By

DANNING W. BLOOM

Bloomington, ^{Ill}Illinois

May 4, 1964

BUCK MEMORIAL LIBRARY
ILLINOIS WESLEYAN UNIVERSITY

ACKNOWLEDGEMENTS

I gratefully acknowledge the advice and assistance of my advisor Mr. R.G. Wilson, chairman of the department of physics, Illinois Wesleyan University. I am also very grateful for the information sent to me by the Science Teaching Center and Professor John King of the Massachusetts Institute of Technology.

TABLE OF CONTENTS

Abstract	page 2
Introduction	page 3
Text	page 4
Conclusion	page 12
Appendix A	page 13
Appendix B	page 16
Appendix C	page 18
Appendix D	page 21
Bibliography	page 23

ABSTRACT

The purpose of this paper was to review information, both experimental and theoretical, concerning the momentum carried by light and its effect on free electrons.

It was theoretically derived that the interaction cross section σ is equal to $8\pi n e^4 / 3m^2 c^4$, where n is the number of electrons, e is the electronic charge in esu, m is the mass of the electron and c is the speed of light.

It was also shown that the interaction can be considered either Thompson scattering or Compton scattering.

It was concluded that the best method of detection of any momentum change is probably that of observing the diffraction pattern of the electron beam.

INTRODUCTION

That light has momentum is a twentieth-century theory which should be demonstrable by showing that it will impart its momentum to electrons. In this paper is given an account of previous research, both theoretical and experimental, that has been done on the interaction between light and free electrons by Hulburt and Breit, Lapp, and Dunn and Ioup.

A hypothetical problem illustrating the magnitudes involved and drawing attention to the sources of error is included. A proof of the agreement of the light quantum and wave theories, a calculation of the cross section of the interaction as well as the number of electrons deflected and the amount of deflection are included as appendixes.

I have also offered other possible methods which may yield more measureable results. In these methods, as in my hypothetical problem, I have used 500 volt electrons and light of wavelength equal to 5000 A.

In 1922, C.J. Lapp, at the University of Illinois, published a report of having produced an observable deflection of a beam of electrons by short electromagnetic waves.¹ The electrons were shot through an intense beam of radiation and were twisted, by means of a magnetic field, into a helix about 70cm long, 3cm in pitch and 1.5cm in diameter. Twisting the beam in such a way resulted in a magnification of any effect. At the far end of the beam was placed a photographic plate which was exposed by the electrons. Comparison of photographs taken when the radiation was off with those taken when it was on indicated a slight scattering effect for radiation with wavelength in the ultra-violet region and a very distinct scattering effect for high energy x-rays.

However, Lapp did no mathematical analysis of the interaction, but only observed that it does occur.

In 1925, E.O. Hulburt and G. Breit published a theoretical report on the momentum imparted to free electrons by radiation.² Assuming that the theories of conservation of energy and momentum hold and that the quantum theory of light and the wave theory of light yield the same interaction cross-section,³ they have shown that the momentum imparted to the electrons is also the same for both theories. They calculated that the ratio of the change in velocity of the electrons, due to the collision with light quanta, to their original velocity is given by $\frac{\Delta v}{v} = \frac{2}{3} \pi \frac{e^4}{m^2 c^4 v^3} \ell \rho$ where ℓ is the distance traveled through radiation of density ρ and m and e are the mass

and charge in electrostatic units of the electron. Thus, even under excellent conditions, they said that $\frac{\Delta v}{v} \approx 10^{-14}$. Consequently, the deflection of the beam would be very hard to detect.

However, if one does not assume, as Hulburt and Breit did, that the interaction is governed by the wave theory, thus causing deflection of the beam as a whole, but that individual electrons are deflected, then a small number of them will each receive a larger deflection. As the number would be very small, this would also be hard to detect. According to Hulburt and Breit, "If an experiment is devised as to have a large number of electrons under observation, there may be a fair theoretical chance of observing deflections."

In 1962, Floyd Dunn and George Ioup, seniors at the Massachusetts Institute of Technology, submitted a report⁴ on the work they had done on the deflection of electrons by light. They had planned on obtaining an observable deflection and then on determining whether the interaction was governed by the wave theory of light or the particle theory of light.

They attempted to deflect a 10 volt beam of electrons with light of 5000 A. They tried to focus magnetically a beam one meter long onto a target .050 inches wide. A beam of light perpendicular to the electron beam was shined on the beam and any electrons that interacted with the photons of light would be deflected enough to miss the target and activate an electron multiplier placed behind the target.

They proved that the interaction between free electrons

and a beam of light can be considered as either Thompson scattering or Compton scattering, since the scattering cross-section is the same for both types of scattering, as is the differential scattering cross section.⁵ Hence, the total momentum imparted by the light to the beam of electrons is the same in either case. The only difference is that in Thompson scattering the momentum is distributed equally among all of the electrons in the beam, where as in Compton scattering, individual electrons receive all of the momentum given up by the photons with which they collide. Thus, the electrons that are not hit receive zero momentum.

Dunn and Ioup were not able to obtain a well-defined, low energy, meter long beam of electrons and as a result were not successful in their experiment. A discussion of why they could not obtain a beam will be included in the section concerning sources of error.

As an indication of the magnitudes involved, a free electron accelerated through a 500 volt potential difference will have a velocity of 1.33×10^7 meters/sec.⁶ It will therefore have a momentum of 1.21×10^{-23} nt-sec.⁷ The momentum of a 5000 A photon, in comparison, is 2.66×10^{-27} nt-sec.⁸ If one photon hits an electron dead center and recoils back upon itself, it will give up twice its momentum to the electron. This momentum will be in the direction of the light beam, that is, perpendicular to the original velocity of the electron. In a distance of one meter, the electron will be deflected a maximum total distance of only .2mm.

Only by using light of much shorter wavelength, hence each photon containing more energy and consequently more momentum, and by decreasing the energy of the electrons and thereby reducing their momentum, will one be able to get a reasonable deflection. Light of wavelength 1750 A and electrons of 50 volts of energy will produce a 2mm deflection.⁹ This would also be very difficult to detect.

An additional factor that increases the difficulty of observing the effect is that only a very small number of electrons are hit per second. Assuming an electron beam of 10 microamperes, 1mm high and 1mm wide, and assuming that one can focus 200 watts of power of light of wavelength equal to 5000 A on a section of the beam 1cm long, the number of photons scattered will only be 156 per second.¹⁰ If each one of these photons hits dead center and recoils back upon itself in an elastic collision, then each electron hit will be given the same momentum and there will be a total deflected current of 156 electrons/second, or 2.5×10^{-17} amperes.¹¹ A current this size would be virtually impossible to detect. Also, this is assuming an ideal situation. The majority of the interactions will not be dead center and therefore the majority of the photons will not recoil back upon themselves. In such a case, they will not impart twice their momentum to the electrons, but only a fraction of it and therefore the deflections produced will not be as large, nor will they all be of the same magnitude.

From the previous information one can easily see that

the momentum of the photons is so small compared to that of the electrons that it will have little effect. In fact, using the method outlined above, the effect would be smaller than the experimental error, unless one used x-rays instead of visible light.

Dunn and Ioup tried to lower the energy of the electrons to 10 volts in order to obtain an observable deflection. But by doing this they increased the sensitivity of the beam. They determined that a transverse electric field of 1 volt per meter would cause a deflection in their beam of 2.5mm. A transverse magnetic field, the size of the earth's magnetic field, will cause the electrons to travel in a circle of 15cm radius. Therefore, even small stray fields will cause the electrons to be deflected more than the light interaction will.¹¹

Dunn and Ioup were never able to produce a well-defined beam one meter long because the earth's magnetic field was distorted by the iron and steel in the walls, tables, ceilings, etc. The electrical equipment used also set up stray fields.

Because of all of these difficulties with simply deflecting a beam of electrons by focusing light on them perpendicularly, I tried to think of other methods of obtaining an observable deflection. One method of observation suggested by Mr. Wilson was to let the beam of electrons be incident on a crystal and look at the diffraction pattern. The change in momentum of the electrons is so extremely small that there would probably be no perceptible smearing of the diffraction pattern. Because of this small change in momentum, the wave-

length of the electrons, equal to Planck's constant, h , divided by the momentum, would change only a fraction of an angstrom. The variation of the velocities of individual electrons in the beam would be greater than that produced by the light and therefore the error in the experiment would exceed the effect we are looking for.¹²

But perhaps if one could produce by magnetic means a beam of electrons that would have a bend in it and then by shining light parallel to the beam but in the opposite direction of the beam, one could change the momenta of a few electrons enough to observe a deflection. If, in such a situation, the photon ideally gave up twice its momentum and recoiled back upon itself, then the change in momentum of the electron would be equal to twice the momentum of the photon. For 500 volt electrons and 5000 A photons, this value would be $\Delta\lambda \approx 1.2 \times 10^{-4}$ A.¹³ This would be an imperceptible change and would doubtless cause no change in the diffraction pattern. Perhaps by using x-rays and lower voltage electrons this method would be feasible. Certainly by shining light opposite to the electron flow there would be more collisions between electrons and photons.

Another idea using the diffraction pattern method would be to simply shine the light perpendicular to the beam of electrons and to see if this would cause a slight shifting or smear of the diffraction pattern due to the few electrons that are deflected. This method appears to be the most promising way of observing the effect, but would probably

not be suitable for a mathematical analysis.

This latter procedure is the one I would have most liked to have tried had I had the means of building the equipment. The electron gun and the crystal would have had to be installed inside a long, evacuated glass tubing. The longer the tubing, the larger the deflection would be and consequently the greater the chance of seeing the effect would be. Therefore, I would have wanted to use a tube at least a meter in length. I was going to build this apparatus using the solder glass technique—a method of fusing, or soldering, glass to glass so as to form a vacuum tight seal. But before I could start on this, a furnace to heat the glass and a saw to cut the glass and leave a very smooth end had to be built, neither of which were finished in time to get started on the apparatus itself.

¹C.J. Lapp, Phys. Rev. 20, 104 (1922).

²E.O. Hulburt and G. Breit, Phys. Rev. 25, 193 (1925).

³Appendix D.

⁴F.E. Dunn and G. Ioup, Deflection of Electrons by Light Pressure, Massachusetts Institute of Technology, 1962.

⁵Appendix D.

⁶Appendix A.

⁷Appendix A.

⁸Ibid.

⁹Ibid.

¹⁰Appendix B.

¹¹Ibid.

¹²Appendix A.

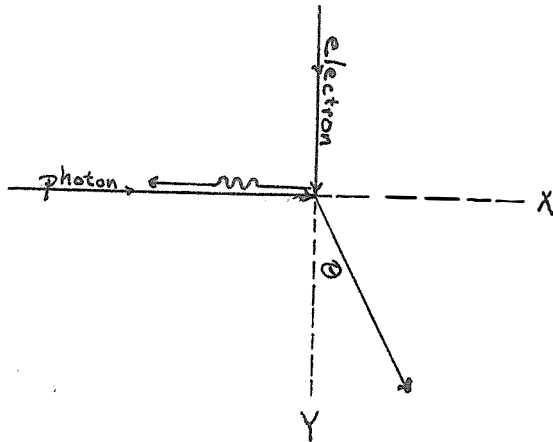
¹³Ibid.

CONCLUSION

This has been a very interesting subject and I feel that if serious work is carried on in this area a method of achieving observation and analysis of this effect will be found. Successful completion of this experiment would result in a proof or disproof of the theory that the interaction is one between photons and electrons, rather than between light waves and electrons. It may also lead to a measurement of the interaction cross section σ .

I wish that I had the time to build the apparatus and continue with the experiment along the lines of the diffraction pattern method because I feel that this could be a very promising procedure.

APPENDIX A: A Photon Impinging upon an Electron and Recoiling Back upon Itself.



From the theory of the conservation of momentum, the total momentum in the X-direction before the collision is equal to that after the collision. Likewise, the total momentum in the Y-direction before the collision equals that after the collision.

$h/\lambda = hf/c =$ the momentum of the photon, where h is Planck's constant of 6.623×10^{-34} joule-sec., λ is the wavelength, 5000 A, f is the frequency $= c/\lambda$, and c is the speed of light.
 $mv =$ momentum of the electron

Since the speed of 500 volt electrons is not relativistic, the problem may be solved by classical methods.

Conservation of momentum:

$$h/\lambda = -h/\lambda + mv_2 \sin\theta \quad \theta \text{ is as shown, } v_2 \text{ is the velocity after collision}$$

$$2h/\lambda = mv_2 \sin\theta$$

$$mv_1 = mv_2 \cos\theta$$

Dividing these two equations we get $\tan\theta = \frac{2h/\lambda}{mv_1}$.

Therefore, the angle at which the electron is deflected is given by $\theta = \tan^{-1} \frac{2h/\lambda}{mv_1}$. The velocity and momentum of a 500 volt electron are found in the following manner:

$$\frac{1}{2}mv_1^2 = 500 \text{ eV} \quad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules} \quad 1 \text{ joule} = \frac{1 \text{ kg} \cdot \text{m}^2}{\text{sec}^2}$$

$$mv_1^2 = (10^3 \text{ eV})(1.6 \times 10^{-19} \text{ j/eV}) = 1.6 \times 10^{-16} \text{ joules}$$

$$\text{mass of the electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$v_1^2 = \frac{1.6 \times 10^{-16} \text{ joules}}{m} = 1.76 \times 10^{14} \text{ m}^2/\text{sec}^2$$

$$v_1 = 1.33 \times 10^7 \text{ m/sec} = \text{the velocity of a 500 volt electron}$$

$$mv_1 = 12.1 \times 10^{-24} \text{ nt-sec.} = \text{the momentum of the electron}$$

before collision.

Now the momentum of the photon must be found.

$$h/\lambda = \text{momentum} \quad \lambda = 10^{-10} \text{ m}$$

$$h/\lambda = \frac{6.623 \times 10^{-34} \text{ j-sec}}{5 \times 10^{-7} \text{ m}} = 1.33 \times 10^{-27} \text{ nt-sec.}$$

$$2h/\lambda = 2.66 \times 10^{-27} \text{ nt-sec.}$$

Thus, the momentum of the electron is around 10^4 times that of the photon. Now we can solve for θ in the above equation.

$$\theta = \tan^{-1} \frac{2.66 \times 10^{-27}}{1.21 \times 10^{-23}} = \tan^{-1} 2.19 \times 10^{-4}.$$

$$\tan \theta = .000219 \quad \theta \cong 0^\circ 1'$$

If $\tan \theta = .0002$, then the total deflection over a distance of one meter will be quite small.

$$\tan \theta = d/10^3 \text{ mm} \quad d = .2 \text{ mm.}$$

A deflection of .2mm will be almost impossible to observe, to say nothing of measuring it. A beam that is fine enough to allow the observation of a .2mm deflection of a few electrons would be very hard, if not impossible, to obtain.

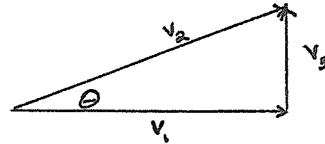
By using electrons accelerated through 50 volts rather than 500 and light of wavelength equal to about 1750 A rather

than 5000 Å, one would theoretically be able to get a deflection of 2mm, which is still quite small. Light of this wavelength is in the ultraviolet region.

The change in momentum of the electrons can be found from $\tan\theta = .000219$.

$$\tan\theta = v_3/v_1 = 2.19 \times 10^{-4}$$

$$v_1 = 1.33 \times 10^7 \text{ m/sec.}$$



$$\sin(1^\circ) \approx \tan(1^\circ) \quad (\text{Standard Math Tables})$$

$\sin\theta = v_3/v_2 \approx v_3/v_1$ Thus $v_2 \approx v_1$ and therefore the change in momentum is extremely small for this type of collision.

For photons parallel to and in the opposite direction of the electron beam, the change in wavelength of the electrons due to the momentum imparted to them by the photons is calculated in the preceding way.

$$\lambda = h/mv \quad mv = \text{momentum of the electrons}$$

$$\frac{d\lambda}{d(mv)} = \frac{-h}{(mv)^2} \quad d\lambda = \left[-h/(mv)^2 \right] d(mv)$$

$$\Delta\lambda = \left[-h/(mv)^2 \right] \Delta(mv) \quad \Delta(mv) = 2 \times \text{momentum of the photon}$$

$$\Delta\lambda = 12 \times 10^{-15} \text{ m} = 1.2 \times 10^{-4} \text{ Å.}$$

APPENDIX B: Number of Electrons Deflected

$\sigma = \frac{8\pi n e^4}{3m^2 c^4}$ σ is called the scattering coefficient and is equal to the fraction of incident photons scattered per cm^3 of irradiated material.¹ The letter n represents the number of electrons in a cubic centimeter. The charge, e, of the electron is in electrostatic units.

To find the number of electrons that are contained in a section of the beam 1mm wide, 1mm high and 1cm in length (volume = 10^{-2}cm^3), the total charge in this volume must be found from the equation $i = \frac{dQ}{dt}$. t = time for the electrons to traverse 1cm.

$$s = vt \quad v = 1.33 \times 10^7 \text{m/sec.} \quad t = s/v \quad s = 10^{-2} \text{m}$$

$$t = 7.52 \times 10^{-10} \text{seconds.}$$

$$Q = it \quad \text{Let } i = 10 \text{ microamps} = 10^{-5} \text{amps.}$$

$$Q = 7.52 \times 10^{-15} \text{ amp-sec} = 7.52 \times 10^{-15} \text{ coulombs.}$$

Each electron carries a charge of 1.6×10^{-19} coulombs.

$$7.52 \times 10^{-15} \text{ coul} / 1.6 \times 10^{-19} \text{ coul/electron} = 4.7 \times 10^4 \text{ electrons.}$$

Thus, there are 4.7×10^4 electrons in this volume of 10^{-2}cm^3 .

$$n = 4.7 \times 10^4 \text{ electrons} / 10^{-2} \text{cm}^3 = 4.7 \times 10^6 \text{ electrons/cm}^3.$$

The fraction of rays scattered = $\sigma = \frac{8\pi n e^4}{3m^2 c^4}$ where
 $e = 4.8 \times 10^{-10}$ esu, $m = 9.1 \times 10^{-28}$ gms, $c = 3 \times 10^{10}$ cm/sec
 and $n = 4.7 \times 10^6$ electrons/cm³.

$\sigma = 3.11 \times 10^{-18}$ This equals the fraction of the light quanta scattered per cm^3 of the beam. In other words, in one cm^3 of the beam, 3.11×10^{-18} times the number of incident photons give up some of their momentum to the electron beam.

Now, how many photons are there incident on each cm^3 , assuming 200 watts are focused on a 10^{-2}cm^3 volume?

200 watts = 200 joule/sec.

Dividing the number of joules per second by the number of joules per photon ($E=hf$) will give the number of photons per second in this volume.

$E = hf = hc/\lambda$ where h is Planck's constant and λ is 5000 Å.

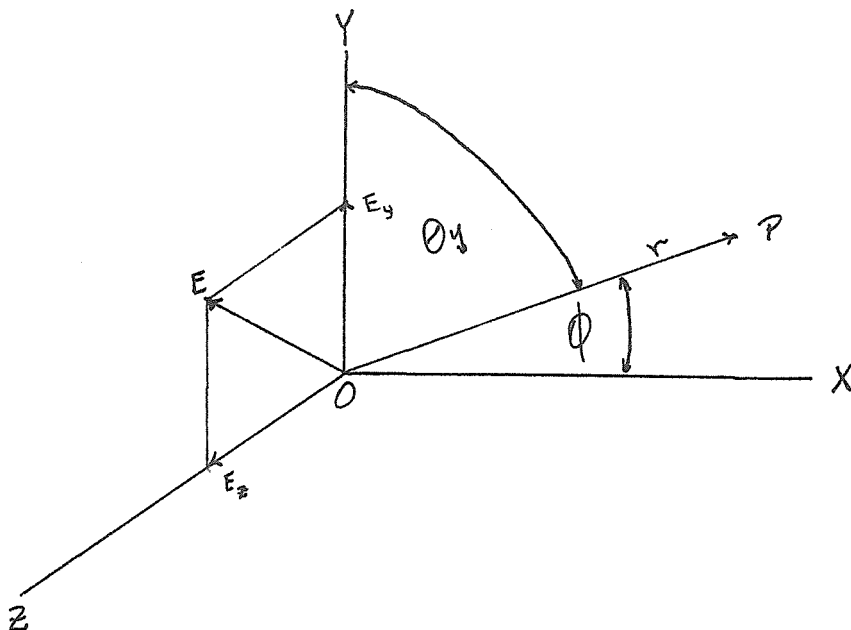
Thus, the energy, E , of each photon is equal to $3.98 \times 10^{-19}\text{j}$.

$$\frac{2 \times 10^2 \text{j/sec}}{3.98 \times 10^{-19} \text{j/photon}} = .5 \times 10^{21} \text{ photons/sec.}$$

There are 5×10^{20} photons/sec incident on this volume. Since 3.11×10^{-18} times this number interact, we have a total of 156 photons scattered each second. The maximum number of electrons scattered per second then is also 156. Multiplying this number by the charge of an electron will give the number of coulombs per second, or amperes, scattered. This gives 2.5×10^{-17} amperes—an extremely small current.

¹Appendix C.

²Appendix A.

APPENDIX C: Cross Section for the Interaction¹

A wave of electric intensity E traverses an electron of charge e and mass m and imparts an acceleration to the electron.

The force F on a charge in an electric field is qE .

$$F = qE \quad q = e \quad F = ma$$

$$ma = eE$$

It can be shown² that $E = \frac{ae}{rc^2} \sin\theta$. $a = \frac{eE}{m}$ and

$$E_{\circ} = \frac{e^2 E}{mrc^2} \sin\theta \quad E_{\circ}/E = \frac{e^2}{mrc^2} \sin\theta$$

Since $I = cE^2/4\pi$, $I \propto E^2$ and $I_{\circ} \propto E_{\circ}^2$

$$\frac{I_{\circ}}{I} = \frac{E_{\circ}^2}{E^2} = \frac{e^4 \sin^2\theta}{m^2 r^2 c^4}$$

Since the direction of E is random in the YOZ plane, E_y is on the average equal to E_z .

$$E_y^2 = E_z^2 \quad \text{and} \quad E_y^2 + E_z^2 = E^2$$

Therefore $E_y^2 = E_z^2 = \frac{1}{2}E^2$ and $I_y = I_z = \frac{1}{2}I$.

$$I_{\circ y}/I_y = \frac{e^4 \sin^2\theta_y}{r^2 m^2 c^4} \quad \text{and} \quad I_{\circ y} = \frac{1}{2} I e^4 \sin^2\theta_y / r^2 m^2 c^4.$$

$\theta + \phi = 90^\circ$, therefore $\sin\theta = \cos\phi$ and $\sin^2\theta = \cos^2\phi$.

$$I_{\phi y} = \frac{1}{2} I_e^4 \frac{\sin^2\phi}{r^2 m^2 c^4}$$

Likewise, $I_{\theta z} = \frac{1}{2} I_e^4 \frac{\sin^2\theta}{r^2 m^2 c^4}$. θ_z is the angle between the ray and the electron's acceleration. $\theta_z = 90^\circ$.

$$I_{\theta z} = \frac{1}{2} I_e^4 / r^2 m^2 c^4.$$

If the primary beam is unpolarized, the intensity of the beam scattered by one electron is $I_e = I_{\theta y} + I_{\theta z}$.

$$I_e = \frac{1}{2} I_e^4 \frac{\sin^2\theta}{r^2 m^2 c^4} + \frac{1}{2} I_e^4 \frac{\sin^2\theta}{r^2 m^2 c^4} = \frac{1}{2} I_e^4 \frac{2}{r^2 m^2 c^4} (1 + \sin^2\theta_y).$$

Since $\sin^2\theta_y = \cos^2\phi_y$, $I_e = \frac{1}{2} I_e^4 (1 + \cos^2\phi_y) / r^2 m^2 c^4$.

If a number of electrons cause scattering, then the intensity of the scattered beam is given by $I_s = n I_e$.

The total power can be found by integrating I_s over the surface of a sphere of radius r .

$$\text{surface of sphere} = \int_0^\pi 2\pi r \sin\phi r d\phi$$

$$P_s = \int_0^\pi I_s 2\pi r^2 \sin\phi d\phi$$

$$P_s = \frac{I_e n e^4}{m^2 c^4} \int_0^\pi (\sin\phi + \sin\phi \cos^2\phi) d\phi$$

$$P_s = \frac{I_e n e^4}{m^2 c^4} \int_0^\pi (\cos\phi + \frac{1}{3} \cos^3\phi) d\phi$$

$$P_s = \frac{8}{3} I_e n e^4 / m^2 c^4.$$

If n equals the number of electrons in a cubic centimeter and I is the energy in the primary beam per cm^2 per second, then the fraction of the primary energy that is scattered per cm of path is $\frac{P_s}{I} = \sigma$.

Thus, $\sigma = \frac{8}{3} n e^4 / m^2 c^4$. σ is called the scattering coefficient and is the cross section of interaction for each electron, where $n = 1$.

¹A.H. Compton and S.K. Allison, X-rays in Theory and Experiment, (D. Van Nostrand Company Inc., New York, 1935) Chap. 3, pp. 117-119.

²Ibid., Appendix II, pp. 774-777.

³Ibid., Chapter 2, p. 57.

⁴Ibid., Chapter 3, p. 118.

APPENDIX D: Agreement of the Light Quantum and Wave Theories

If it can be shown that Thompson's theory of scattering leads to the same results as Compton's theory of scattering, then it can be assumed that there is agreement between the wave theory of light and the particle theory of light in this case.

Thompson scattering will be dealt with first. If $n = 1$, then $\sigma = \frac{8\pi e^4}{3 m^2 c^4} \cdot I$.¹ But r_0 , the classical radius of the electron, is equal to e^2/mc^2 , where e is in esu units.²

$$\text{Now } \sigma = \frac{8}{3} \pi r_0^2 \cdot I$$

$$P = \frac{dW}{dt} = I n r_0^2 \int_0^\pi (1 + \cos^2 \phi) \sin \phi d\phi.$$

But $\pi \int_0^\pi \sin \phi d\phi = -\pi \cos \phi \Big|_0^\pi = +2\pi$. Therefore, the $\int \pi \sin \phi d\phi$ equals $\frac{1}{2}\Omega$ and $\pi \sin \phi d\phi = \frac{1}{2}d\Omega$, where $\Omega = 4\pi$ is a solid angle.

$$\text{Thus } P = \frac{dW}{dt} = \frac{1}{2} I n r_0^2 \int_0^\pi (1 + \cos^2 \phi) d\Omega.$$

But P is also equal to σI .³

$$P = \sigma I = \frac{1}{2} I n r_0^2 \int_0^\pi (1 + \cos^2 \phi) d\Omega, \quad n = 1$$

$$\text{and } \sigma = r_0^2 \int_0^\pi (1 + \cos^2 \phi) d\Omega / 2. \quad \text{Therefore } \frac{d\sigma}{d\Omega} = r_0^2 \frac{(1 + \cos^2 \phi)}{2}.$$

It can be shown⁴ that for Compton scattering the Klein-

Nishina formula will give the following:

$$d\sigma = \frac{1}{2} r_0^2 d\Omega \frac{1 + \cos^2 \phi}{1 + E(1 - \cos \phi)^2} \left[1 + \frac{E^2 (1 - \cos \phi)^2}{(1 + \cos^2 \phi)(1 + E(1 - \cos \phi))} \right]$$

where $E = \frac{hf}{m_0 c^2}$. For $E \ll 1$, $\frac{d\sigma}{d\Omega} = r_0^2 \frac{(1 + \cos^2 \phi)}{2}$.

Thus, $\frac{d\sigma}{d\Omega}$, the differential scattering cross-section, is the same for Compton scattering, a particle interaction, and Thompson scattering, a wave interaction. It is also possible to show that σ , the scattering cross section, is the same for both theories.

Thus, there is agreement between the wave and particle

theories of light. Therefore we are justified in using either approach.

¹Appendix C.

²W.K. Panofsky and M. Phillips, Classical Electricity and Magnetism (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), Chapter 21, p. 325.

³Appendix C.

⁴L.C. Yuan and C.S. Wu, Nuclear Physics (Academic Press, New York, 1961), Vol. 5, part A, Chapter 1.1, p. 81.

BIBLIOGRAPHY

- Bohm, David, Quantum Theory, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1951.
- Compton, A.H. and Allison, S.K., X-rays in Theory and Experiment, New York, D. Van Nostrand Co., 1935.
- Dunn, F.E. and Ioup, G., "Deflection of Electrons by Light Pressure," issued by the Science Teaching Center, Massachusetts Institute of Technology, 1962.
- Hughes, A.L., and DuBridge, L.E., Photoelectric Phenomena, New York, McGraw-Hill Book Company, 1932.
- Hulburt, E.O. and Breit, G., "On the Momentum Imparted to Electrons by Radiation," Physical Review, volume 25, 1925, p. 193.
- Lapp, C.J. "The Effect of Short Electromagnetic Waves on a Beam of Electrons," Physical Review, volume 20, 1922, p. 104.
- McConnell, J., Quantum Particle Dynamics, Amsterdam, North Holland Publishing Co., 1958.
- Michels, W.C. and Patterson, A.L., Elements of Modern Physics, New York, D. Van Nostrand Co., 1951.
- Panofsky, W.K. and Phillips, M., Classical Electricity and Magnetism, Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1955.
- Semat, Henry, Introduction to Atomic and Nuclear Physics, New York, Holt, Rinehart and Winston.
- Slater, J.C., Modern Physics, New York, McGraw-Hill Book Company, 1955.
- Yuan, L.C. and Wu, C.S., Methods of Experimental Physics -- Nuclear Physics, New York, Academic Press, 1961.