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Fun, Games & Economics: An appraisal of game theory in economics

Jamus Jerome Lim

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by

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"My work is a game, a very serious game."

M.C. Escher

Abstract: *Game theory has had a profound influence on many fields of the social sciences since its rise to prominence more than fifty years ago. This paper provides an overview of the main concepts in game theory and studies four main areas of its application in economic problems – oligopolistic competition, externalities & public goods, market equilibrium and general equilibrium. The conclusion is that game theory has found a natural place in economics and will continue to contribute to it for many years to come.*

I. Introduction

Game Theory – better described as 'Interactive Decision Theory' – was first thrown into the spotlight in 1944 with the publication of von Neumann and Morgenstern's *Theory of Games and Economic Behaviour*. Although previous authors have been credited with the discovery of game theory (Aumann 1987; Dimand & Dimand 1992; Fudenberg & Tirole 1991), von Neumann and Morgenstern firmly entrenched game theory in the realm of economics by providing a whole new way of looking at the competitive process, through the eyes of strategic interactions between economic agents.

This paper will aim to provide an appraisal of game theory in the field of economics. In section II, a historical overview of the development of game theory in economics is explored and this is followed by a quick and intuitive discussion of the central concepts in game theory (section III). Section IV, the central focus of the paper, will discuss the many ways in which game theory has been applied in economics, and criticisms and future direction is touched on in section V. Section VI concludes the discussion.

II. Historical overview

The first studies of games in the economics literature were the papers by Cournot (1838), Bertrand (1883) and Edgeworth (1897) on oligopoly pricing and production. Borel then gave the first reasonably systematic treatment of game theory in 1921, although he did not provide any rigorous proofs for his speculations. This was built upon by von Neumann who went on to prove a central concept – the minimax theorem – in 1928, before collaborating with Morgenstern to publish the *Theory of Games* in 1944 (Leonard 1992).

Despite the initial excitement following the publication, game theory spent a long period in the doldrums, as economists were slow to see the importance of the theory. Game theory was pigeonholed as a theory for the small numbers case in economics, and its popularity waned (Schotter & Schwödiauer 1980). However, in the 1950s game theory enjoyed a revival for three reasons: first, the ascendancy of mathematical economics; second, the widening of the field of game theory as it was applied to the analysis of general equilibria; and third, the strong interest of the military (Leonard 1992; Schotter & Schwödiauer 1980).

From then on, game theory grew from strength to strength. Its continued growth into the 1970s in terms of both theoretical extensions as well as widened applications in other areas of economics proved that game theory was here to stay (Aumann 1987). The award of the Nobel Prize in Economics to three game theorists in 1994 sealed the intimate relationship between economics and game theory.

III. A Brief Introduction to Game Theory

Although space constraints make it impossible to provide a thorough exploration of all the concepts of game theory, this section will attempt to cover the most important ones[1]. The discussion will proceed by identifying the various forms and types of games, followed by the main solution concepts developed to solve games.

3.1 Describing games

Games can be represented in three forms, each providing a different level of detail; the form applied will depend on the type of analysis to be carried out. The *extensive* or *tree form* gives a very detailed representation of the game. It consists of a complete formal description of how the game is played, the sequence in which the players move, what

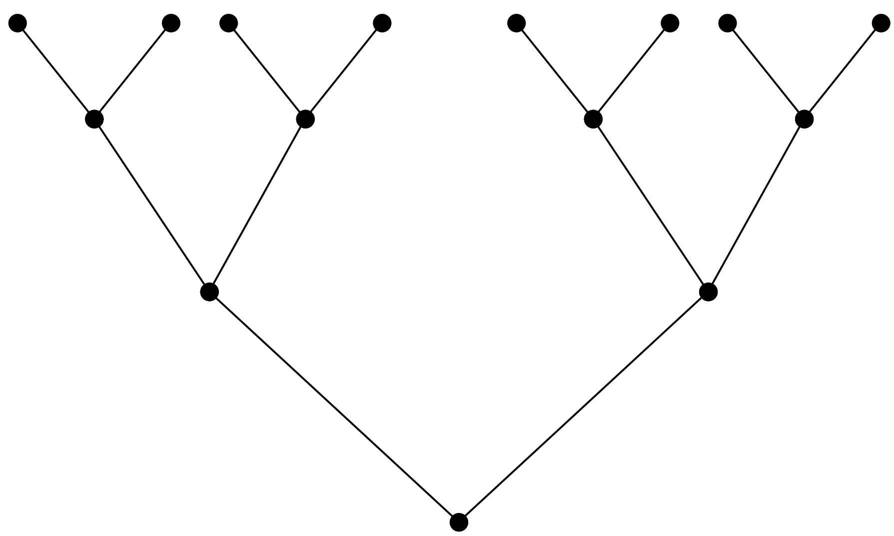
[1] For a detailed explanation on the mathematical theory of games, see Luce & Raiffa's (1957) text *Games and Decisions* or Fundenberg & Tirole's (1991) text *Game Theory*. For an excellent non-mathematical introduction to game theory, see Zagare's (1984) paper *Game Theory: Concepts and Applications*.

they know at the times they must move, how chance occurrences enter the picture, and the payoffs to each player (Aumann 1987).

A less stringent description of the rules of the game and the strategies and payoffs available to each player is given by the *strategic* or *normal form* of a game. This form appears as a matrix giving the strategies of players, but it loses a good deal of the descriptive richness (Zagare 1984).

The *coalitional* or *characteristic function form* of a game is used when minimal information is required to describe the game. It provides information about the payoffs to a single player or coalition and little else (Aumann 1987; Schotter & Schwödiauer 1980). A graphical representation of each of these three forms is provided in *Figure 1*.

Extensive/Tree form



Strategic/Matrix form

		Player 2	
		F_1	***F_2***
Player 1	***P_1***	2, 2	4, 1

P₂	1, 4	3, 3

Coalitional/Characteristic function form

$$v_i(s) = max \left\{ \Sigma \lambda^I x^I, x \in V(S) \right\}$$

where x represents an outcome,

v the value of the game &

λ an arbitrary weight

Figure 1

Games can also be classified into different types. Whether a game is *cooperative* and *non-cooperative* would depend on whether the players can communicate with one another. Where players can cooperate and binding agreements made, the game is known as a cooperative game. If communication or contracting is ruled out, the game becomes a non-cooperative one (Schotter & Schwödiauer 1980). Although cooperative games provide an avenue for cooperation, it should be noted that players may very well choose *not* to cooperate (Bacharach 1976).

Zero-sum and *non-zero-sum games* are used to describe games where there is the possibility of a conflict of interest. In a zero-sum game, the interests of the players are directly opposed, and there are no common interests. More likely, however, are non-zero-sum games where shared interests exist (Fudenberg & Tirole 1991; Luce & Raiffa 1957; Zagare 1984). Although zero-sum games may be criticised as unrealistic (Bacharach 1976), Riker (1962, p. 31) has argued that so long as a game is “perceived as requiring indivisible victory, the zero-sum model is probably best”.

Finally, games have also been categorised into *2-person* or *n-person games*, where *n* involves three or more players. Such a distinction is important as such games are both quantitatively and qualitatively different, and are due to the introduction of the dynamics of coalition formation between two or more players (Luce & Raiffa 1957; Zagare 1984). As such, the approach towards n-person games is vastly different from that taken towards 2-person games.

3.2 Solution concepts

A solution concept is a function that "associates outcomes, or sets of outcomes, with games" (Aumann 1987, p. 464) and usually provides the payoffs that the outcome(s) yield to players. When a solution is required for a game, it is necessary to first specify the form and type of the game under investigation.

Non-cooperative solutions involve solving for the non-cooperative or Nash equilibrium point of the game. This basic non-cooperative solution concept may be applied to games in both normal and strategic form, and provides a solution where each player maximises his payoff given the other players' strategies (Katz & Rosen 1994; Nash 1950). Therefore, for all players i, a strategy choice $s*$ must exist such that

$$P_i\,(s*) \geq P_i\,(s* \,|\, s_i) \qquad \text{for all } s_i \in S_i$$

where P_i represents the payoffs and s_i the strategy choice.

Non-cooperative equilibria encompass narrower solution concepts such as Zermelo's theorem – which states that a finite game of perfect information has a pure-strategy Nash equilibrium (Zermelo 1913; Kuhn 1953) – as well as von Neumann's famous (1928) minimax theorem – that there is always a solution to zero-sum games, as well as many others.

Cooperative solutions are designed to capture the stable outcome of a bargaining problem. Cooperative solution concepts rest on the ideas of imputations, which are the sets of payoff combinations that satisfies both individual and group rationality; and domination, when an outcome can be achieved and is preferred to another by all members in a coalition (Aumann 1987; Zagare 1984).

The core of a game is the set of outcomes that is not dominated by any coalition (Luce & Raiffa 1957); in a game v where the payoff that a coalition can expect is $v(S)$, the core is the set of feasible outcomes x which satisfies

$$Co(v) = \sum x_i \geq v(S)$$

It should be noted that the core may be empty, that is, no core exists; this consequently limits its usefulness, since many games have an empty core (Rubinstein 1979; Zagare 1984).

The nucleolus, $Nu(v)$, is another solution set that chooses the set of outcomes that minimises the maximum domination of any coalition. This solution concept is more appealing than the core as it has been proven that the nucleolus is always non-empty and has only a single point (Schmeidler 1969).

The von Neumann and Morgenstern solution concept is more sophisticated and complicated than the previous two methods described. It provides a solution to the game, $NM(v)$, where the set of imputations satisfies two conditions: first, 'internal consistency', meaning that any two imputations in $NM(v)$ must not dominate each other; second, 'external protection', that any imputation *not* belonging to $NM(v)$ must be dominated by some imputation *in* $NM(v)$ (von Neumann & Morgenstern 1947).

The *value solution* – applied to cooperative games – is the unique outcome that results when all players behave rationally. It can be thought of as a "reasonable compromise or arbitrated outcome, given the power of the players" (Aumann 1987, p. 469). This single solution is the Shapley value,

$$\phi(v)_i = \sum (n - k)!(k - 1)! \,/\, n! \bullet [v(K) - v(K - i)]$$

where $v(K) - v(K - i)$ is the marginal contribution of player i to coalition K and $(n - k)!(k - 1)! \,/\, n!$ is the probability that in a random formation of coalition K, player i will be the player joining the coalition. The Shapley value can be interpreted as a solution concept where each player is giving his or her marginal contribution (Aumann 1987; Shapley 1953).

IV. Economic Applications of Game Theory

Game theory, being concerned with the behaviour of decision makers and their interactions, seems to have limited applicability in economics, where the *homo economicus* makes decisions that depend primarily on his own choices in a known way (Hargreaves Heap *et al.* 1992). However, as economics moves towards maturity, it has had to develop new techniques for the description and prediction of rational choice under non-ideal assumptions. Game theory fills this gap perfectly, and is therefore the "natural approach to the economics of uncertainty" (Borch 1968, p. 150).

The scope of influence of game theory in economics is considerably wide. Each sub-section here will present an economic problem, followed by its game-theoretic interpretation, and possible solutions. Space permits only a detailed examination of game theory in the areas of oligopolistic competition, externalities & public goods, market equilibrium and general equilibrium. Other areas only briefly mentioned are those of the allocation of joint costs, the development of incentive-compatible mechanisms and the development of voting mechanisms[2].

4.1 Oligopolistic competition

Since an oligopoly, by definition, involves a mutual dependence of firms and recognition of this interdependence (Katz & Rosen 1994), game theory is therefore easily applied to oligopolistic decision making. This section will provide a study of, first, static oligopoly models and second, dynamic oligopoly models[3].

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[2] Specialised literature covering these fascinating areas are available, notably Shubik (1962), Hurwicz (1960; 1972; 1973), Gibbard (1973) and Satterthwaite (1975). A quick review can be found in Schotter & Schwödiauer (1980).

[3] Friedman's (1977) book provides an extensive study of game theory in oligopolies, while Katz & Rosen's textbook provides a good, non-mathematical summary of Cournot, Bertrand and Edgeworth's contributions, pitched at a moderate level.

Static equilibrium models are best understood by a discussion of a Cournot duopoly, where each firm's strategy involves a variation of output level. Let price be given by $f(Q)$ (price being a function of total quantity Q), total production by q and production costs by C. The profits for each firm is therefore

$$\pi_i = f(Q)(q_i) - C(q_i) \quad \text{where } i = 1, 2$$

The Cournot equilibrium is represented as

$$\pi_i\,(q_c) \geq \pi_i\,(q_1 | q_2)$$

The vector $(q_1 | q_2)$ reflects the interdependence of the firms' actions, and no single firm can have greater profits by selecting an output level other than q_c (Friedman 1990; Malinvaud 1972).

Figure 2 presents a geometric representation of the Cournot equilibrium. The Cournot equilibrium q_c occurs at the intersection of the reaction curves for each firm, shown as $q_1^*(q_2)$ and $q_2^*(q_1)$, where a reaction curve shows the decision maker's best strategy, given the strategy of the other firm.

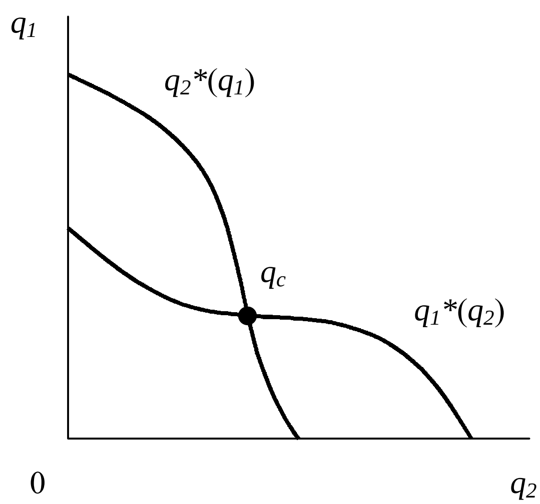

Figure 2 [4]

[4] Diagram adapted from Fudenberg & Tirole (1991)

The Cournot equilibrium is a solution concept within the more general Nash equilibrium, and the duopoly model can be extended to study other forms of Nash equilibria. The Bertrand equilibrium involves firms varying their price level instead of output; the Stackleberg equilibrium involves a 'leader' and a 'follower' firm, where the leader maximises profit subject to the follower choosing according to his reaction function; and the Hotelling solution involves firms choosing their location along a line (Forges & Thisse 1992; Hotelling 1929; Sonnenschein 1987).

Dynamic equilibrium models involve firms and consumers meeting repeatedly under identical circumstances. Consider two firms, labelled 1 and 2. Each firm has three strategies, to produce 'low', 'middle' or 'high' quantities of output. The payoffs are represented in the matrix labelled *Figure 3*.

		Firm 2		
		L	***M***	***H***
Firm 1	***L***	15, 15	5, 21	5, 10
	M	21, 5	12, 12	2, 5
	H	10, 3	5, 2	0, 0

Figure 3[5]

The combinations (*L, L*), (*M, M*) and (*H, H*) may be thought of as the monopoly, Cournot and competitive outcomes, respectively. (*M, M*) would be the unique Nash equilibrium in a single period game. However, strategies in repeated games are more complicated than those employed in a single period game, due to the ability of firms to 'punish' or 'reward' other firms; as a result, a more collusive outcome such as (*L, L*) may be attained. This equilibrium outcome of mutual benefit is the distinct advantage of repeated games (Friedman 1971).

[5] Example adapted from Sonnenschein (1987).

Such oligopolistic games can readily be applied to studies of actual games played by firms. For example, General Motors, after invading the low-price car market in 1921 with the Chevrolet, engaged a two-player zero-sum game between themselves and Ford, then the market leader with the Model T. Applying a duopolistic game situation, General Motors played out an imaginary game which guided its actual strategies. The result was that by the end of 1927 Ford had lost its market dominance to General Motors[6] (McDonald 1975).

4.2 Externalities & Public Goods

The problems of externalities can be broadly classified into positive and negative externalities. This section will address both forms – and their game theoretic solutions – by way of example, illustrating negative externalities through a simple pollution problem and positive externalities through a public goods problem[7].

A traditional solution to the negative externality problem has been to assign property rights, creating a situation where a bargaining process arises involving negotiation for taxes and/or subsidies (Coase 1960). The problem thus becomes one of determining imputations in the core for an *n*-person cooperative game (Schotter & Schwödiauer 1980).

Assume two firms, one being a factory, and the other being a farm. Externalities are assumed to be reciprocal, meaning that pollution by the factory influences the farm and pollution by the farm influences the factory (not unbelievable – factory waste could affect crops, and farm pesticides could damage manufactured produce). The initial payoff matrix for the two agents, *without* externalities, is shown below in *Figure 4*. Assuming

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[6] 'The Game of Business' is the title of McDonald's (1975) book that details many case studies of oligopolistic games as well as other forms of games as applied to business. In fourteen chapters McDonald guides the reader through different types of games actually played in real-world situations.

[7] Examples in this section were adapted from Bacharach (1976), Fudenberg & Tirole (1991) and Friedman (1990).

that they have two strategies, produce low (*L*) or produce high (*H*), each player chooses independently and payoffs are maximised at (*H*, *H*), which is the dominant strategy for each agent.

		Factory	
		L	***H***
Farm	***L***	9, 5	9, 8
	H	16, 5	16, 8

Figure 4

However, amending the payoffs to take into account externalities, we obtain a new payoff matrix, as shown in *Figure 5*.

		Factory	
		L	***H***
Farm	***L***	6, 2	-3, 5
	H	13, -7	4, -4

Figure 5

Although the dominant strategy for each firm remains as (*H*, *H*), it is no longer the Pareto-optimal solution. The result is a prisoner's dilemma problem, and both firms stand to gain by slowing production to *L* each, but will not do so. This "lack of co-operation results in a 'jointly irrational' outcome" (Bacharach 1976, p. 75). A solution to this problem would be to devise taxes and/or subsidies that will induce the choice of (*L*, *L*) for the firms, creating a new non-cooperative, Pareto-optimal equilibrium.

Before moving on to discuss public goods, it is interesting to note that a more complicated problem of a similar vein did arise in Maine in the late 1960s, the principal players being oil companies, local businesses and local residents (McDonald 1975).

In the study of public goods, the main problem is the 'free rider' problem, where an individual "refrains from taking costly action because he knows that someone else will take it" (Katz & Rosen 1994, p. 234). The problem therefore becomes a non-cooperative game of incomplete information.

Consider two players, $i = 1, 2$. Players decide simultaneously to contribute to a public good, and if at least one of them contributes both players will derive a benefit of 1 and if neither does they derive a benefit of 0. Each player's cost of contributing is given by c_i, and the payoff matrix is shown in *Figure 6*.

		Player 2: **Contribute**	Player 2: **Don't**
Player 1	**Contribute**	$1 - c_1, 1 - c_2$	$1 - c_1, 1$
	Don't	$1, 1 - c_2$	0, 0

Figure 6

The payoffs to player i, given the strategy of each player s_i, is given by

$$P_i(s) = \max(s_1, s_2) - c_i s_i$$

The Bayesian Nash equilibrium is a pair of strategies (s_1^*, s_2^*) such that for each player i and for every possible value of c_i, the strategy $s_i^*(c_i)$ will maximise expected payoffs.

Experimental evidence has supported the game-theoretic analysis of the public goods problem in the real world. Palfrey and Rosenthal studied this problem in the University of California, Los Angeles in 1989, utilising two players and a single public good, and yielded similar results.

4.3 Market equilibrium

One of the more significant breakthroughs of game theory has been the description of market equilibria in an n-person economy as a game. This involves describing market exchanges as that of a cooperative game[8].

The game-theoretic approach to multilateral exchange provides an alternative, better developed theory of price determination, and although different in style of analysis from traditional demand-and-supply theory, provides results consistent with it (Bacharach 1976). The discussion will begin with a simplified 1-seller, 2-buyer model, before describing extensions to allow a larger, m seller-n buyer model.

Consider a single seller, A, and two buyers, B_1 and B_2. A is willing to sell for a (or more) units of money while the buyers are willing to pay b_i (i = 1, 2) or less. Letting (x, y_1, y_2) denote the respective payoffs, the following equations may be derived.

$$x \geq a, \qquad y_1 \geq 0, \qquad y_2 \geq 0 \tag{1}$$

$$x + y_1 \geq \max(a, b_1), \quad x + y_2 \geq \max(a, b_2), \quad y_1 + y_2 \geq 0 \tag{2}$$

$$x + y_1 + y_2 = \max(a, b_1, b_2) \tag{3}$$

Assume $a < b_1 < b_2$ (with little loss of generality). This would imply that equation (3) becomes

$$x + y_1 + y_2 = b_2 \tag{4}$$

Substituting $x + y_2 \geq b_2$ from equation (2) we obtain (since $y_1 \geq 0$ from (1))

$$y_1 = 0 \tag{5}$$

Taking $x + y_1 \geq b_1$ from equation (2) and the result $y_1 = 0$ we obtain

$$x \geq b_1 \tag{6}$$

---

[8] Examples in this section were adapted from Bacharach (1976) and Schotter & Schwödiauer (1980).

Since $y_2 \geq 0$ (also from (1)), taking the result $y_1 = 0$ and substituting into (4) gives

$$x \leq b_2 \tag{7}$$

The final three equations (5)-(7) completely represent the core solution concept. The payoffs to the core represent the price, and it falls between b_1 and b_2. The core solution, therefore, determines the market outcome (Bacharach 1976).

Geometrically, the core corresponds to the range *AB* in the supply-demand diagram shown in *Figure 7*.

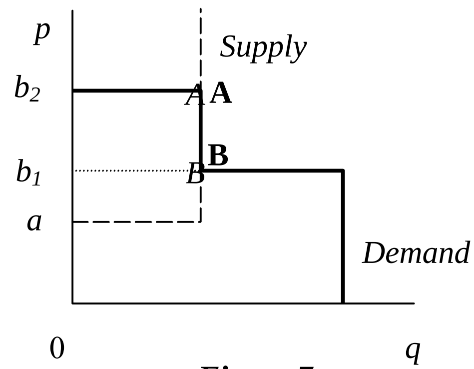


Figure 7

This example can easily be extended to deal with a case of *m* sellers and *n* buyers. The Pareto-efficient core can once again be calculated. A schematic representation of the core of an economy with A_m sellers and B_n buyers will yield price *p*, which falls between the highest supply price a_k and the lowest demand price b_{n-k+1}. *Figure 8* illustrates this result.

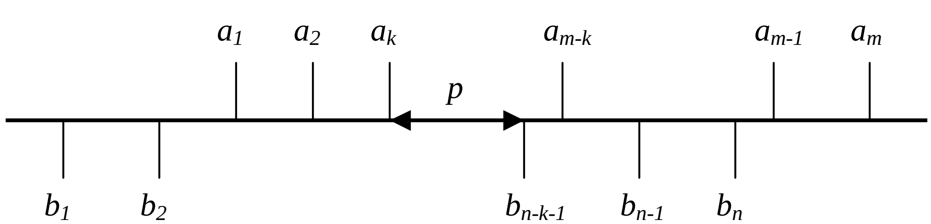


Figure 8

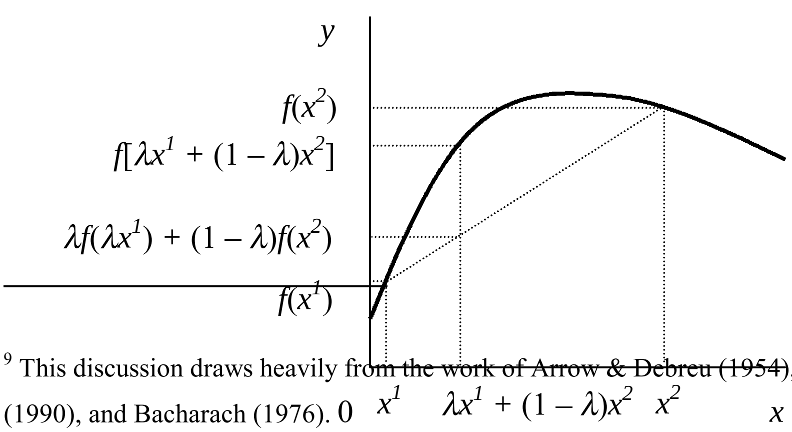
The primary advantage of utilising a game representation of markets lies in the fact that markets that do not fulfil some of the neoclassical assumptions (such as convexity of preferences or absence of externalities) are still amenable to analysis (Schotter & Schwödiauer 1980).

4.4 General equilibrium

This section will move on to describe general Walrasian equilibrium in a competitive exchange economy as a non-cooperative Nash equilibrium in a game of complete information. The assumptions here are that agents know the rationing scheme and send market trade offers which, given other agents' trade offers, result in a payoff of preferred final transactions for each agent[9]. Three general conditions must be fulfilled before to prove that competitive equilibrium is a Nash equilibrium. They are, for *i* players:

> *Condition 1*. The payoff of player *i* is continuous in all strategies and concave in the strategy of player *i*; and the demand for a good tends to infinity as the price of the good tends to zero.
>
> *Condition 2*. The strategy set of an individual player is chosen so as to maximise utility subject to his or her budget constraint.
>
> *Condition 3*. The strategy sets of other players are chosen such that no alternative agreement would yield higher payoffs simultaneously for all players.

In short, condition 1 ensures market clearing, while the other two ensure that the Limit Theorem – that trades are efficient and utility maximising – holds (Friedman 1990). *Figure 9* and *10* illustrate conditions 1 and 2 & 3, in a 2 player-2 good context.



[9] This discussion draws heavily from the work of Arrow & Debreu (1954), as described in Friedman (1990), and Bacharach (1976).

Figure 9

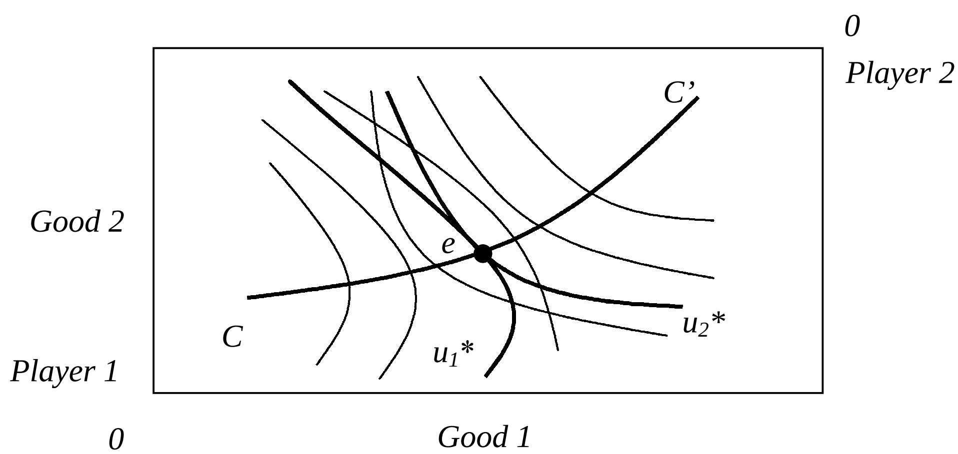


Figure 10

There are certain advantages of utilising a game representation of general equilibrium. First, modelling the problem in the form of a game provides the ability to distinguish between feasible and equilibrium actions; and second, it allows the introduction of institutional arrangements into the analysis of general equilibria (Schotter & Schwödiauer 1980).

4.5 Other applications

A few other applications of game theory within the field of economics merit brief mention. Game theory has helped in the allocation of joint costs, providing attractive core, nucleolus and value solutions. Practically, this has been applied to the study of pricing for public utilities such as sewage and water in several municipalities in Hungary (Bogardi & Szidarovsky 1976).

Game theory has also been applied to the development of incentive-compatible mechanisms and institutions, used to improve allocative efficiency; as well as strategy proof voting mechanisms, to ensure non-dictatorial and Pareto-optimal Nash outcomes (Schotter & Schwödiauer 1980).

V. Criticisms & Future Direction

Criticisms have been levied against game theory, and two in particular are worth noting. First, game theory is sometimes judged to have failed, because some games possess solutions and others do not, and those that do have solutions sometimes provide too many. Here Bacharach (1976) argues that the failure of game theory to give unambiguous solutions to certain classes of games does not imply flaws or inadequacies in the theory, just that it is the nature of things. We cannot expect a unique, rational answer all the time in a complex, possibly irrational world. Second, there is the argument that game theory contorts the way individuals view the world, as one of selfish interactions. However, game theory in itself "has no moral content, makes no moral recommendations, is ethically neutral" (Aumann 1987, p. 497). It *studies* selfishness, it does not *recommend* it.

However, on the whole the future of game theory is bright. Recent developments of game theory in economics include the discovery of new Nash equilibria (such as the Markov equilibrium used to study the common resource problem); refinement of equilibrium concepts (such as the stability concept used to study signalling in markets) and evolutionary game theory, an exciting new field studying social conditioning and behaviour selection (Cho & Kreps 1987; Levhari & Mirman 1980; Weibull 1995).

VI. Conclusion

Just as mathematics has carved itself a niche in economics – making it seem unthinkable that a modern economist isn't at least familiar with basic mathematical economics – game theory has also found itself with a definite role to play in economic analysis. Instead of the limited neoclassical analysis that is tied to only one institutional framework – competitive markets – game theory offers a more flexible analysis of economic problems that includes institutions as well. Its increasing pervasiveness in basic textbooks is a testament to this fact (Bacharach 1976, Schotter & Schwödiauer 1980).

References

Aumann, R.J. 1987, ‘Game theory’, in ***The New Palgrave: A Dictionary of Economics, vol. 2***, ed. Eatwell, J., Milgate, M. & Newman, P., Macmillan, London, England

Arrow, K.J. & Debreu, G. 1954, ‘Existence of an equilibrium for a competitive economy’, ***Econometrica***, vol. 22, pp. 265-290

Bacharach, M. 1976, ***Economics and the Theory of Games***, Macmillan, London, England

Bertrand, J. 1883, ‘Théorie mathématique de la richesse sociale’, ***Journal des Savants***, pp. 499-508

Bogardi, I. & Szidarovsky, F. 1976, ‘Application of game theory in water management’, ***Applied Mathematical Modelling***, vol. 1, no. 1, pp. 11-20

Borch, K.H. 1968, ***The Economics of Uncertainty***, Princeton University Press, Princeton, New Jersey

Cho, I.K. & Kreps, D.M. 1987, ‘Signalling games and stable equilibria’, ***Quarterly Journal of Economics***, no. 102, pp. 179-221

Coase, R.H. 1960, ‘The problems of social cost’, ***Journal of Law and Economics***, no.3, pp. 1-44

Cournot, A.A. 1838, ***Researches sue les Principes Mathematiques de la Theorie des Richesses,*** Libraire de Sciences Politiques et Sociale, M. Riviere & cie, Paris

Dimand, R.W. & Dimand, M.A. 1992, ‘The early history of the theory of strategic games from Waldegrave to Borel’, in ***Toward a History of Game Theory***, ed. Weintraub, E.R., Duke University Press, Durham, England

Edgeworth, F. 1881, ‘La teoria pura del monopolio’, ***Giornale degli Economisti***, pp. 13-31

Forges, F. & Thisse, J.F. 1992, ‘Game theory and industrial economics: an introduction’ in ***The New Industrial Economics***, ed. Norman, G. & La Manna, M., Edward Elgar, Aldershot, England

Friedman, J.W. 1971, ‘A non-cooperative equilibrium of supergames’, ***Review of Economic Studies***, vol. 38, pp. 1-12

Friedman, J.W. 1977, ***Oligopoly and the Theory of Games***, North-Holland, Amsterdam, Holland

Friedman, J.W. 1990, ***Game Theory with Applications to Economics, 2nd edition***, Oxford University Press, New York, New York

Fudenberg, D. & Tirole, J. 1996, ***Game Theory***, MIT Press, Cambridge, Massachusetts

Hargreaves Heap, S., Hollis, M., Lyons, B., Sugden, R. & Weale, A. 1992, ***The Theory of Choice: A Critical Guide***, Blackwell, Oxford, England

Hotelling, H. 1929, ‘Stability in competition’, ***Economic Journal***, vol. 29, pp. 41-57

Hurwicz, L. 1960, ‘Optimality and informational efficiency in resource allocation process’ in ***Mathematical Methods in the Social Sciences***, ed. Arrow, K.J., Karlin, S. & Suppes, P., Stanford University Press, Stanford, California

Hurwicz, L. 1972, ‘On informationally decentralised systems’ in ***Decision and Organisation***, ed. McGuire, C.B. & Radner, R., North-Holland, Amsterdam, Holland

Hurwicz, L. 1973, ‘The design of mechanisms for resource allocation’, ***American Economic Review***, vol. 63, no. 2, pp. 1-30

Katz, M.L. & Rosen, H.S. 1994, ***Microeconomics, 2nd edition***, Irwin, Burr Ridge, Illinois

Kuhn, H. 1953, ‘Extensive games and the problem of information’, ***Annals of Mathematics Studies***, Princeton University Press, vol. 28

Leonard, R.J. 1992, 'Creating a context for game theory' in ***Toward a History of Game Theory***, ed. Weintraub, E.R., Duke University Press, Durham, England

Levhari, D. & Mirman, L. 1980, 'The great fish war', ***Bell Journal of Economics***, pp. 322-344

Luce, R.D. & Raiffa, H. 1957, ***Games and Decisions***, John Wiley & Sons, New York, New York

Malinvaud, E. 1972, ***Lectures on Microeconomic Theory***, North-Holland, Amsterdam, Holland

McDonald, J. 1975, ***The Game of Business***, Doubleday & Company, Garden City, New York

Nash, J.F. 'Equilibrium points in n-person games', ***Proceedings of the National Academy of Sciences of the U.S.A.***, vol. 36, pp. 48-49

Palfrey, T. & Rosenthal, H. 1989, ***Underestimated probabilities that others free ride: an experimental test***, mimeograph, University of California, Los Angeles, California

Riker, W.H. 1962, ***The Theory of Political Coalitions***, Yale University Press, New Haven, Connecticut

Rubinstein, A. 1979, 'A note about the 'nowhere denseness' of societies having an equilibrium under a majority rule', ***Econometrica***, vol. 47, pp. 511-514

Schmeidler, D. 1969, 'The nucleolus of a characteristic function form game', ***Journal of Applied Mathematics***, vol. 17, no. 6, pp. 1163-1170

Schotter, A. & Schwödiauer, G. 1980, 'Economics and the theory of games: a survey', ***Journal of Economic Literature***, vol. 18, no. 2, pp. 479-527

Shapley, L. 1953, 'A value for n-person games', ***Annals of Mathematics Studies***, vol. 28, pp. 307-317

Shubik, M. 1962, 'Incentives, decentralised control, the assignment of joint costs and internal pricing', ***Management Science***, vol. 8, no. 3, pp. 325-343

Sonnenschein, H. 1987, 'Oligopoly and game theory', in ***The New Palgrave: A Dictionary of Economics***, ed. Eatwell, J., Milgate, M. & Newman, P., Macmillan, London, England

von Neumann, J. 1928, 'Zue theorie des gesellschaftsspiele', ***Mathematische Annalen***, no. 100, pp. 295-320

von Neumann, J. & Morgenstern, O. 1944, ***Theory of Games and Economic Behaviour***, Princeton University Press, Princeton, New Jersey

von Neumann, J. & Morgenstern, O. 1947, ***Theory of Games and Economic Behaviour, 2nd edition***, Princeton University Press, Princeton, New Jersey

Weibull, J.W. 1995, ***Evolutionary Game Theory***, MIT Press, Cambridge, Massachusetts

Zagare, F.C. 1984, 'Game theory: concepts and applications', ***Sage University Paper series on Quantitative Applications in the Social Sciences***, series no. 07-041, Sage Publications, Newbury Park, California

Zermelo, E. 1913, ***Über eine Anwendung der Mengenlehre auf der Theorie des Schachspiels***, in Proceedings of the Fifth International Congress on Mathematics