



4-16-1997

The Economic Rationale of a Multi-State Lotto

Brett Roush '97

Illinois Wesleyan University

Follow this and additional works at: https://digitalcommons.iwu.edu/econ_honproj



Part of the [Economics Commons](#)

Recommended Citation

Roush '97, Brett, "The Economic Rationale of a Multi-State Lotto" (1997). *Honors Projects*. 67.

https://digitalcommons.iwu.edu/econ_honproj/67

This Article is protected by copyright and/or related rights. It has been brought to you by Digital Commons @ IWU with permission from the rights-holder(s). You are free to use this material in any way that is permitted by the copyright and related rights legislation that applies to your use. For other uses you need to obtain permission from the rights-holder(s) directly, unless additional rights are indicated by a Creative Commons license in the record and/ or on the work itself. This material has been accepted for inclusion by faculty at Illinois Wesleyan University. For more information, please contact digitalcommons@iwu.edu.

©Copyright is owned by the author of this document.

**The Economic Rationale of a
Multi-State Lotto**

**Brett Roush, Department of Economics, IWU
Advanced Research Seminar / Research Honors
Final Draft: 16 April 1997**

I. INTRODUCTION

State lotteries generated over \$34 billion in gross sales in FY 1996, which resulted in over \$11 billion of contributions to state funds (Meyers, 1996). This nominal sales level is about 30 times that of twenty years ago. Obviously, enormous growth has occurred in the last two decades; this growth alone makes lotteries interesting to economists. However, state lotteries possess other characteristics that have made them the object of numerous studies.

First, lotteries offer an opportunity to study how consumers react to straightforward risky situations. Other forms of risk-taking--investing in the stock market, for example--or even many forms of gambling--handicapping horses, for example--do not offer the simplistic situation posed by lotteries. These instruments involve nothing more than a small payment followed by a purely random determination of winner and losers. The potential winnings, moreover, can range from the nugatory (a free ticket) to the awe-inspiring (in July of 1993 the Powerball jackpot reached \$110 million). Besides gaining insights into consumer behavior in risky situations, lottery research is important for public policy issues. Lottery profits are donated to state funds, so predicting the revenue-generating capability of a state's portfolio of games helps budgetary planners. Similarly, knowing what factors affect revenue generation will benefit the state as a whole. Finally, another aspect of public policy often studied is where the lottery--which can be considered a voluntary transfer of wealth from the state's residents to the state--receives its funds. In particular, is this voluntary tax skewed towards the poor in terms of who contributes?

The purpose of this study is to examine a relatively recent lottery product--the multi-state lotto--from an economic perspective. Drawing upon economic theory as well as past studies this paper first seeks to understand how a consumer acts under risk, then examines lotteries from a demand-side perspective in order to more exactly explain the motivations behind adoption of a multi-state lotto. Finally, this paper employs a pooled time-series regression in order to analyze how timely and appropriate Illinois' adoption of a multi-state lotto ("The Big Game") is and to predict its sales in the coming years.

Section II provides a history of the lottery in America, as well as necessary terminology. Section III delineates the theory and related literature on the demand for lottery products and develops hypotheses. Section IV describes the empirical model used to test the hypotheses, and Section V displays the results. Finally, Section VI gives concluding remarks.

II. BACKGROUND

Lotteries have their own terminology. Table 1 provides a brief description of the major terms used in this paper, as well as insight concerning the differences among the most common games. The terminology is the same as that used by Mikesell and Zorn (1987).

Table 1: Lottery Terminology

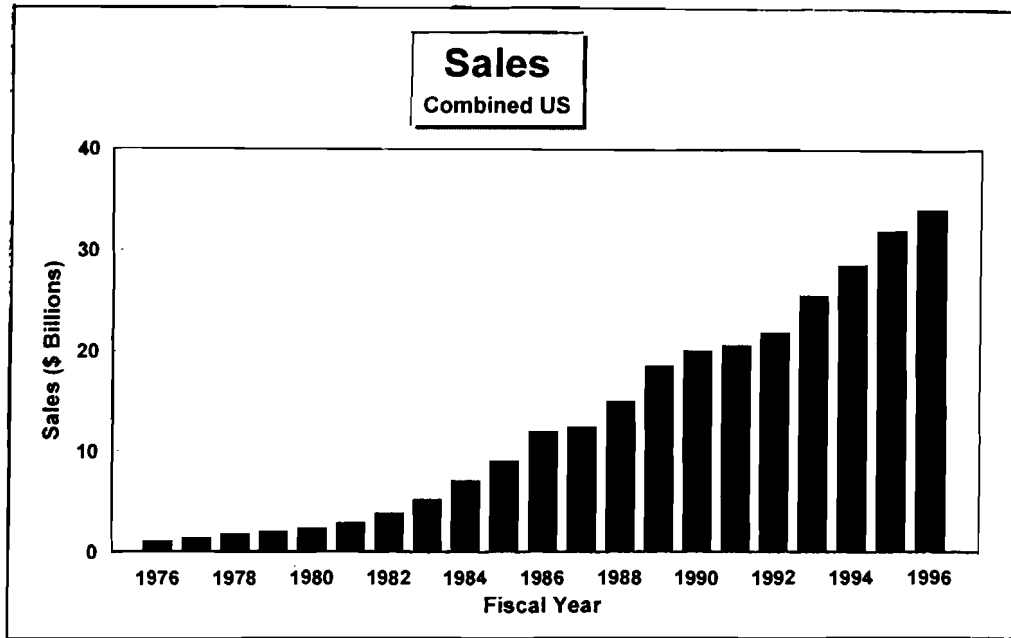
Portfolio	The set of games offered by a state.
Passive	A game in which periodic drawings are held to determine winners of predetermined prizes; the player buys tickets as in a raffle and does not specify his numbers (as in some other games).
Instant	A game in which a ticket has an opaque coating that when removed (usually by scratching it off) reveals instantly whether the player has won a prize.
Numbers	This is similar to the illegal game of the same name. It requires that the player choose numbers, normally 3 or 4, each from the set zero through nine. Drawings for predetermined prizes are usually held daily.
Lotto	The player selects a certain number of numbers from a given range. A 6/54 game would require choosing 6 different numbers from the set 1-54 inclusive. Prizes are awarded for matching at least a certain number of the drawing's numbers; the jackpot is often pari-mutuel and large--usually \$1 million or more. Being pari-mutuel means the number of tickets bought as well as the number of winners affect the jackpot's size. If the jackpot is not won, it is rolled over into the next drawing.
Multi-State	A game in which more than one state sells tickets. Most multi-state games are the "lotto" type.

Lotteries have contributed to public causes in North America since colonial times, with such notable figures as George Washington and Thomas Jefferson supporting them. However, a scandalous affair with the Louisiana Lottery Company in the late 1800s caused lotteries to be outlawed until the modern lottery began in 1963, in New Hampshire. A complete history is unnecessary for the purpose of this paper; please see *Illinois* (1987) and Scott (1973) for more detailed depictions of early lotteries in America.

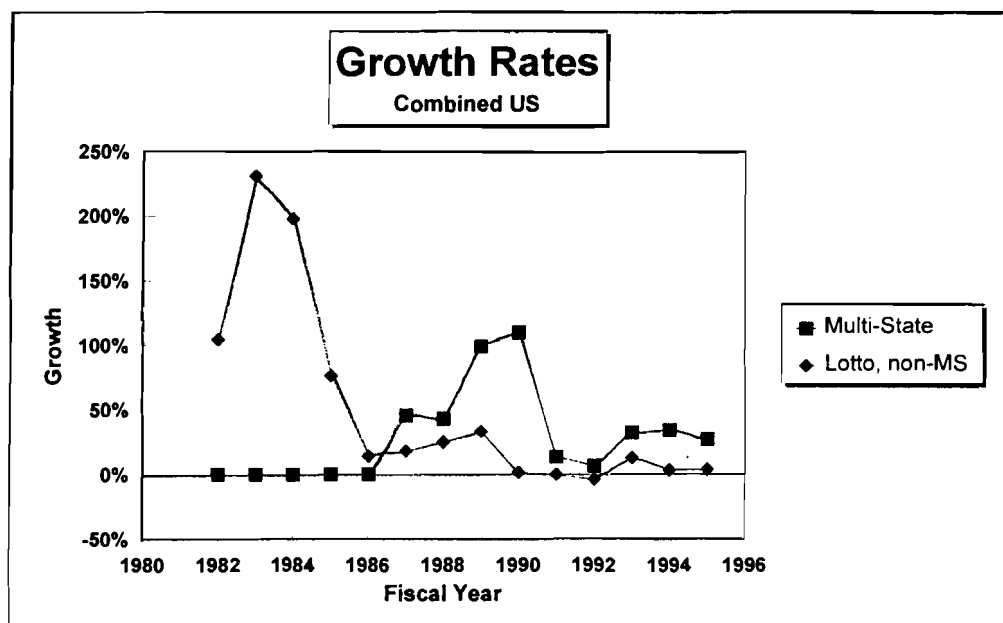
Since the modern lottery's inception in 1963 a few of its historical aspects do merit attention. Early games were quite dissimilar--New York, New Jersey, and New Hampshire all offered passive games, but New York's tickets cost \$3, New Jersey's \$0.50, and New Hampshire's \$1. Moreover, payout rates as well as drawing frequencies varied widely (Clotfelter and Cook, 1989). In the three decades since these first passive games were introduced lotteries have evolved to become nearly homogenous. First, and most importantly, passive games are virtually extinct, having been replaced by instant games where consumers have the option of choosing their own numbers. Also, though many details differ among games, each state offers a nearly identical product mix consisting of instant games, some form of daily numbers game, and a large-jackpot lotto. Of the 38 states offering some type of lottery product, all offer instant games, 37 also offer a lotto product, and 30 offer instant games, lotto, and a numbers game. Such similarity in product types and portfolios is important to this analysis. When pooling states, having generally similar state lottery environments augments the validity of the empirical analysis.

States have almost universally found financial success with this variety of games. Figure 1 graphically depicts the impressive rise in lottery sales since 1976. This success has, in turn, led to enormous growth in the number of states offering lotteries. Figure 2 shows how states have jumped onto the lottery bandwagon over the last two decades.

Figure 1: US Lottery Sales since 1981

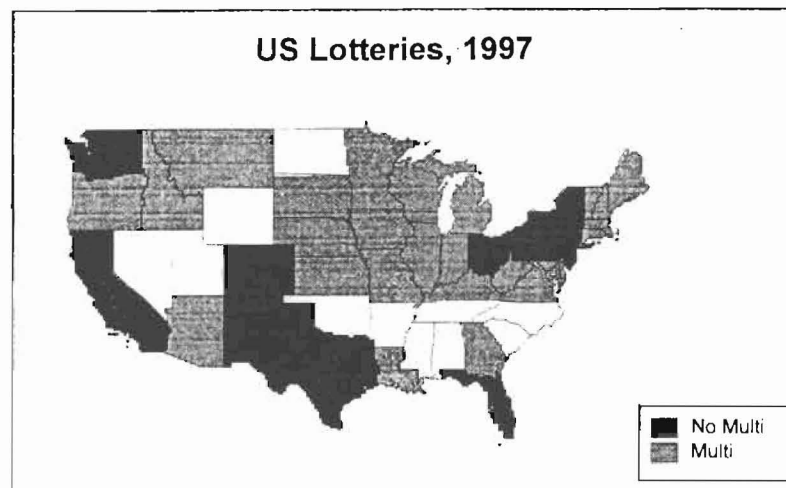
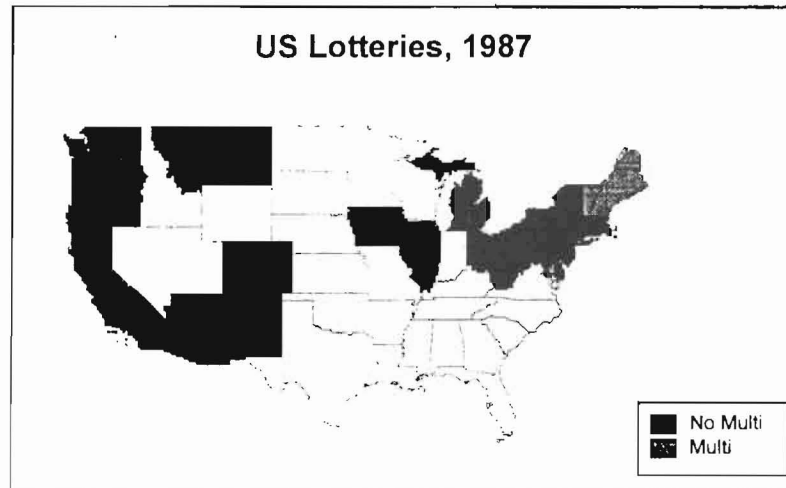
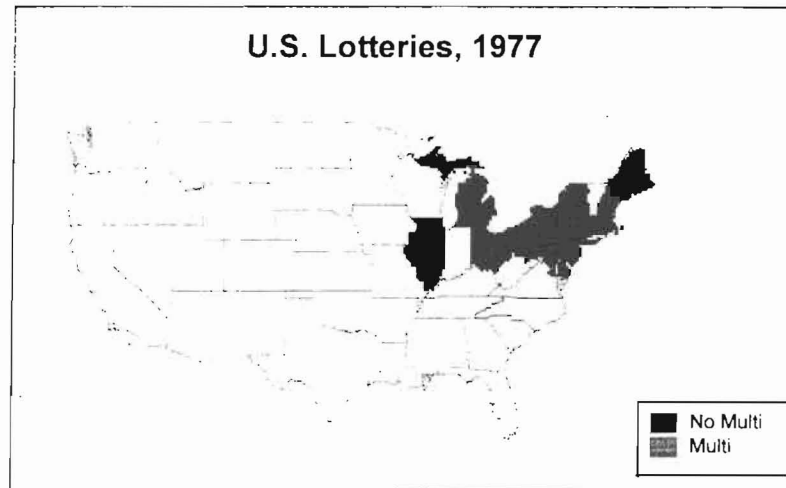


Source: La Fleur's '96 World Lottery Almanac



Source: La Fleur's '96 World Lottery Almanac

Figure 2: The Spread of Lotteries in America



Source for all three: La Fleur's '96 World Lottery Almanac

A notable addition to possible lottery products occurred in September of 1985 when New Hampshire, Vermont, and Maine joined to create Tri-State Megabucks. This game was the first modern lottery product to cross state lines. Though having longer odds of winning, the game allowed a potential jackpot much greater than any of the three small New England states could offer on their own. In the 12 years since this initial multi-state lottery a number of others have begun. Lotto*America was created in 1988 with 7 states joining together; now called Powerball, it includes 20 states. Also, three Western states have united in the Tri-West games. Most recently, August 31, 1996 marked the inception of The Big Game, a 6-state lotto formed by Illinois, Michigan, Georgia (which defected from Powerball), Virginia, Maryland, and Massachusetts. This latest multi-state game means that nearly 50% of people in states with lotteries have access to multi-state games.

Looking back at Figures 1 and 2, the role of multi-state lotto games becomes apparent. The latter figure shows how quickly this game type has been adopted--going from 3 to 28 states in just 10 years. Moreover, the "growth" graph in Figure 1 depicts that lotto growth rates stabilized by 1986. Just as revenue from single-state lottos flattened, multi-state games entered a period of high growth. The graph does not tell the whole story, either, since the six states making up "The Big Game" will cause another spike in the growth rate of multi-state lotto games. As Clotfelter and Cook (1993) state, multi-state lottos are attractive because "for the game of lotto, bigger is better." Indeed, it appears that multi-state lottos are the next step in a state lottery portfolio.

Already the economic rationale of a multi-state lotto begins to show itself. We must now look into the theory of the consumer as well as the salient characteristics of lotto products in order to more fully understand a multi-state lotto's role.

III. THEORY AND LITERATURE

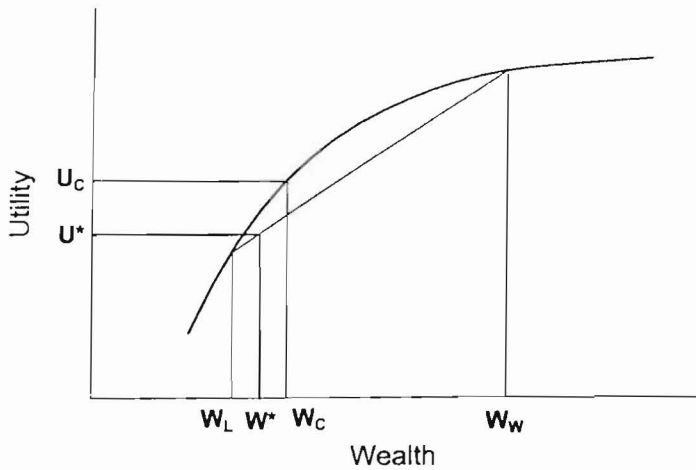
A basic supply and demand model will facilitate answering the two questions of this paper: why would a state adopt a multi-state lotto, and what is the predicted revenue for

Illinois in The Big Game. Vrooman (1976) explains that "the equation which is estimated represents the demand function for regular lottery tickets because the independent variables used in the equation do not influence the supply of tickets." Indeed, the supply of lotto tickets is essentially a horizontal line at the price--a state will sell any number of tickets at the given price level¹. Thus this paper will concentrate on elements of the demand curve. Two sets of variables must be examined. First, the change being considered is a state's adoption of a new product. Attention must therefore be given to those variables affected (e.g. the jackpot size and the fact that the product is new). Second, this study analyzes lotteries from a statewide perspective, across borders and time. It must therefore also consider those demographics variables reflecting state differences. Before examining individual variables, though, this paper must develop a general model of the consumer acting in a risky situation.

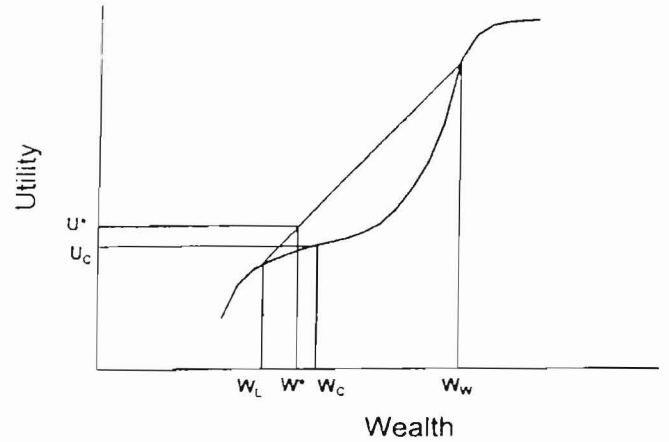
A. The Consumer and Risk

Demand for gambling appears superficially to contradict the economic notion of a rational and risk-averse consumer. Solely due to the diminishing marginal utility of wealth consumers ought to avoid a gamble--and they should doubly avoid the lottery. Not only are their expected utilities lower than their existing utilities, but the expected value of a lottery is negative--consumers stand to actually lose wealth. Milton Friedman and L.J. Savage, in their classic 1948 article "The Utility Analysis of Choices Involving Risk," explore the seeming paradox of consumers who have been observed to buy insurance (risk-averse behavior) as well as gamble (risk-seeking behavior). Their main hypothesis states that the total utility of wealth curve has a convex portion where a change in socioeconomic class occurs. A consumer's utility increases at an increasing rate around the perceived border between, for example, being in the lower middle income class and being firmly within the middle income class. How does this imply that consumers would play a lottery? The following explanation answers this, assuming a simplified lottery in which only one prize is offered.

Figure 3: Two Utility of Wealth Curves



Case 1: Strictly Concave



Case 2: Concave and Convex

Figure 3 presents two possible total utility of wealth curves; Case 1 shows the most common shape used by economists, while Case 2 portrays the one hypothesized by Friedman-Savage. As shown in each case, a person has current wealth W_C . A gamble offers the possibility of winning a prize such that the consumer's wealth would equal W_W , while costing an amount such that the consumer would have a wealth of W_L if he were to lose. In both Case 1 and Case 2 the straight line connecting the utilities associated with W_W and W_L represents all possible expected utilities of the gamble. This line consists of $p[U(W_W)] + (1-p)[U(W_L)]$ for every p from 0 to 1, with p being the probability of winning the lottery and $U(W_X)$ being the utility of wealth W_X .

Lotteries have negative expected values², so every possible expected value lies between W_L and W_C . In Case 1, this quality combined with the fact that the marginal utility of wealth (the slope of the consumer's total utility curve) constantly decreases means that every expected utility is less than U_C . W^* and its corresponding U^* serve as an example. W^* lies between W_L and W_C , therefore U^* is below current utility U_C . Since the expected utility is less than the current utility the consumer would not participate in the lottery.

In Case 2, the convex portion of the curve represents *increasing* marginal utility of wealth. Under the assumption of strictly decreasing marginal utility of wealth all expected

utilities of the lottery are below U_C . In the Friedman-Savage model, though, we see that at some expected wealth levels lower than W_C the expected utility is *above* U_C . This quality means that a lottery could have a negative expected value, yet still induce a person to play. W^* and U^* illustrate where the expected wealth is lower than the current wealth, yet the expected utility exceeds current utility. The Case 2 consumer would play this lottery.

Such a reversal of implications from the standard utility model results from the enormous possible gains in utility experienced at the change in socioeconomic class. So, the concave, then convex curve presented by Friedman and Savage (1948) predicts observed gambling behaviors. But does it make sense? They explain the intuitive appeal of their model using a fair game as an example: "an unskilled worker may prefer the certainty of an income about the same as that of the majority of unskilled workers to an actuarially fair gamble that at best would make him one of the most prosperous unskilled workers . . . yet he may jump at an actuarially fair gamble that offers a small chance of lifting him . . . into the 'middle' or 'upper' class." Such an argument obviously carries over into games that are *not* actuarially fair, such as lotteries.

Two noteworthy responses to the Friedman-Savage model followed its publication. First, Markowitz (1952) presents a modified version of the variously concave and convex curve, having uncovered some inconsistencies with observed reality in the Friedman-Savage proposal. His modification of their curve's shape is not applicable to this study, but his concept of customary wealth does clarify some issues. Markowitz depicts consumers, under most circumstances, as possessing some normal, "customary" level of wealth. Regardless of what level this is, consumers will consider themselves to be far from a change in socioeconomic class--in other words, consumers are firmly within the concave portion of their utility curves. Under the Friedman-Savage model a consumer could be near an increase in socioeconomic class. This consumer would then accept a gamble that is essentially a lottery's opposite--one offering a probable small gain and an improbable but devastating loss, one so dramatic as to cause a drop in socioeconomic class. Such gambles are infrequently accepted.

Markowitz's idea precludes such a predicted action by not allowing consumers, customarily, to see themselves as being on the brink of entering a new socioeconomic class. His idea also means that changes in socioeconomic class occur at different wealth levels for different consumers.

In terms of the graphical presentation, Markowitz's idea causes the customary wealth (or current wealth, as it was referred to before), W_c , to always occur in a concave section of the total utility curve. Moreover, consumers then have a "classical" section of diminishing marginal utility following their customary wealth before experiencing increasing marginal utility. On the figures presented before, having the vertical (utility) axis intersect the horizontal (wealth) axis at the level of customary wealth incorporates this idea (not shown). The initial concave section of the utility curve would then intersect this vertical axis.

Kwang (1965) presents a model of utility that both accounts for the shortcomings in the Friedman-Savage model and solidifies the conceptual "change in socioeconomic class." His model, shown in Figure 4, is briefly summarized here. A noticeable deviation from Friedman and Savage and Markowitz is that the marginal utility of wealth constantly decreases. However, non-differentiable sections exist at indivisibility points. In essence, Kwang interprets a change in socioeconomic class to be a level of wealth which permits the purchase of some indivisible good with cheaper substitutes that are not nearly as desirable. For example, "a person may choose between having a Volkswagen and having a Mercedes." A quantum jump in utility occurs at each point where some indivisible and desired good becomes obtainable.

An advantage of Kwang's model over both the Friedman-Savage one and Markowitz's is that it shows repeated deviations from classical diminishing marginal utility. Friedman and Savage allude to the possibility of multiple inflection points but do not develop it. They state that "At the moment, there seems to be no observed behavior that requires the introduction of additional convex segments . . . it [may] be necessary to add such segments." Markowitz, though enhancing the Friedman-Savage model, limits the utility function. He decides that "[it]

has *three* inflection points [emphasis added]." Including multiple indivisibility points depicts consumers as having many different levels for utility jumps, which fits well with the typical description of consumers having unlimited wants.

Figure 4: Kwang's Utility of Wealth Curve

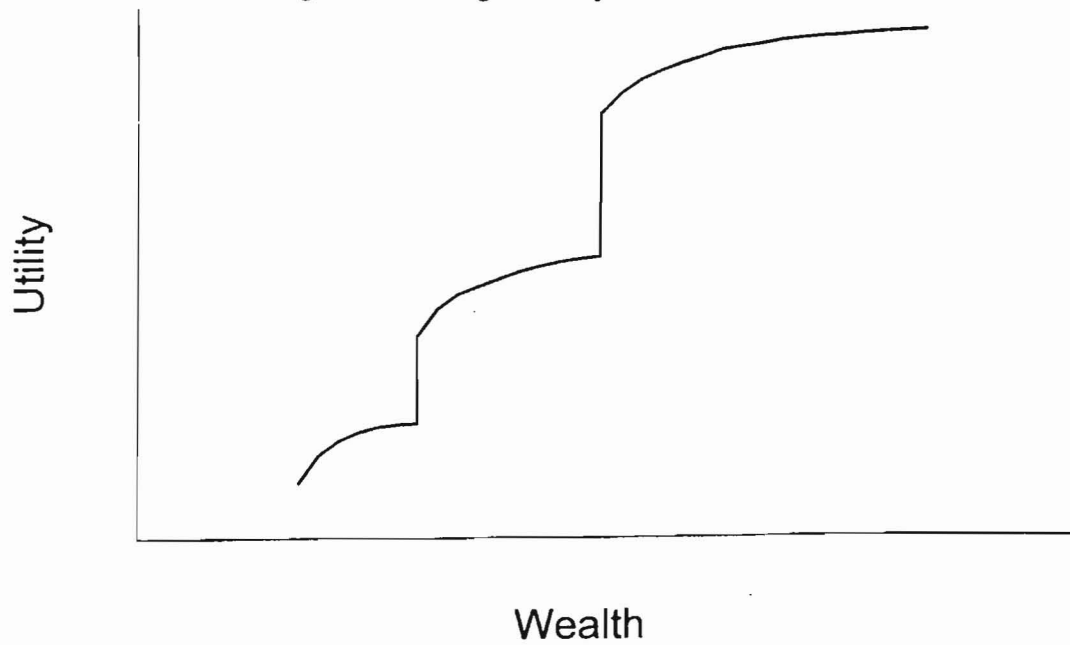


Figure 5: A Composite Model of the Utility of Wealth Function

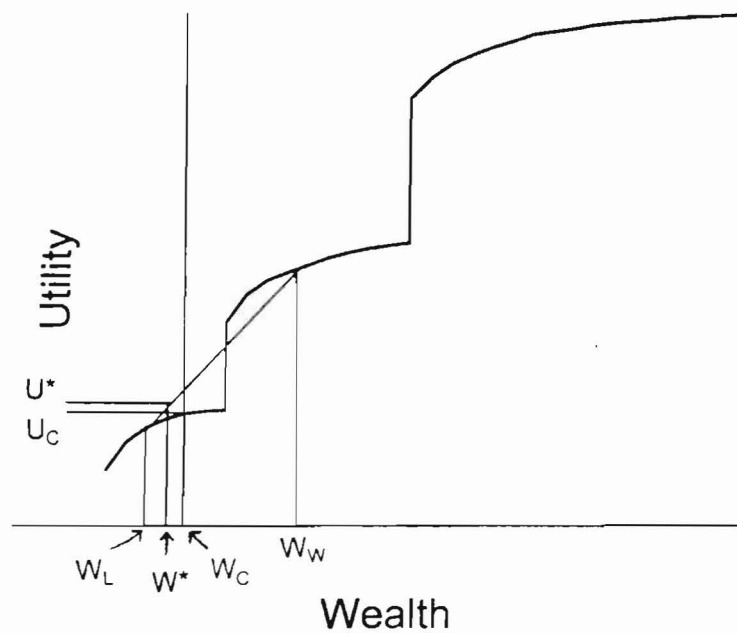


Figure 5 combines Kwang's always concave but at times non-differentiable utility function, Markowitz's idea of customary wealth, and the graphical analysis illustrated on the basic Friedman-Savage model presented earlier. A feature of the shown utility curve is increasing differentials between wealth levels where indivisibility points occur. Kwang does not include this feature in his introduction of the curve. However, it seems reasonable to present indivisible goods as having greater absolute differences in wealth levels necessary for obtaining them rather than decreasing relative differences³. Using this model of consumer behavior under risk, hypotheses can be generated concerning lottos and how changes in relevant variables will affect sales.

B. Lotto Variables

1. Prizes

The most obvious and publicized change that introducing a multi-state lotto presents is a new prize structure--and an increased jackpot. How does a different prize structure affect sales? Three measurements are introduced by R. Clay Sprowls (1970). These are the game's expected value, the probability of winning any prize, and the Gini coefficient--a measure of the prizes' inequality. Comparing these values for the New York Lotto and some foreign lotteries he determines that a large (but not enormous) grand prize coupled with numerous small prizes leads to a high-sales game. Vrooman (1976) tests these concepts on New York and elicits unsatisfactory results. None of the prize structure variables are significant, and an increase in the probability of winning a prize *decreases* sales.

Following these unsatisfactory results research into prizes' effects on sales has focused on the jackpot. This makes sense--the jackpot is the number advertised; winners are featured by the media; players dream of what they'll do if they win the big prize. In short, focusing on the jackpot means focusing on the same aspect of the prize structure as the players.

Figure 6: Jackpot Increase and the Total Utility of Wealth

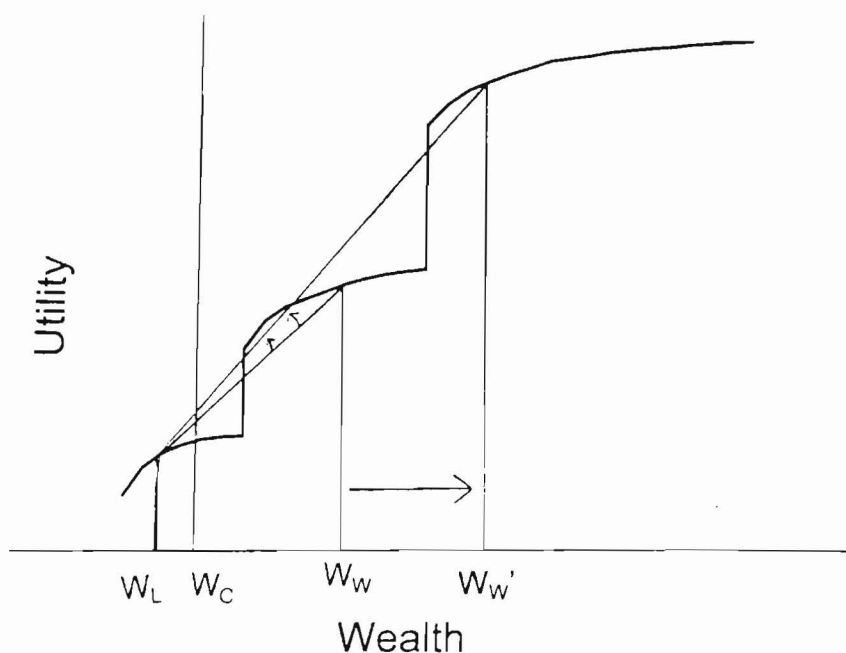


Figure 6 shows how a jackpot increase may alter the model of the consumer presented above. The possible loss from a ticket is usually the same or experiences a minimal increase, so the position of W_L in relation to W_C is essentially unchanged. However, the rightward shift in W_W can be quite extreme, in the millions of dollars. Such an increase has two distinct effects on a consumer's propensity to play.

First, a greater number of indivisibility points may be crossed by the higher potential jackpot. This would result in a possible expected utilities line that, at all points, is higher than the previous one (depicted in Figure 6). Obviously, when the expected utility from the lotto increases, *ceteris paribus*, a consumer would be more likely to play it. However, there is no guarantee that more indivisibility points are crossed. A jackpot increase may simply move the potential winning wealth further along the same concave portion of the total utility curve. This would result in a new expected utilities line that is *lower* at every point than the one at the smaller jackpot level. In this case, *ceteris paribus*, the consumer would be less likely to play. A second effect from a jackpot increase is that for a greater number of consumers the potential

winning wealth crosses an indivisibility point. For some consumers who previously would not play the expected utility of the lotto now exceeds their current utilities.

The model of the consumer under risk has flaws by allowing for the extremely counter-intuitive prediction that an increase in the jackpot could decrease demand. However, the other effects predicted--higher demand from indivisibility points crossed for both consumers who played previously and those who did not--do make sense. The author believes that these effects will dominate the one causing a decrease in demand, and therefore an increased jackpot will cause higher demand.

Empirical studies have repeatedly found this predicted relationship, with higher jackpots associated with higher revenues. This relationship is most strongly found when lotto sales are examined on a weekly basis. In these cases a series of rollovers can drastically increase the jackpot. DeBoer (1990) finds an increasing elasticity of sales with respect to jackpots in the New York Lotto from 1985 to 1988. Similarly, Thiel (1991) analyzes the Washington State Lotto and finds that, given a change in the jackpot, the revenue increase is greater when the change occurs from a higher jackpot level.

A 1993 study by Clotfelter and Cook using annual data supports the relationship between jackpot size and demand indirectly. The authors find a positive relationship between the population and the revenue per capita for lottos across states. In other words, as population increases, the amount each person plays in a lotto also increases. Population is a proxy for jackpot size in a pari-mutuel game such as lotto--the jackpot depends partially on the amount of sales. Clotfelter and Cook observe that "in contrast to lotto, there is no obvious reason why per capita sales of the numbers game will be influenced by the scale of the game." Indeed, their regression for the fixed-jackpot numbers games finds no such statistically significant relationship between population and revenue per capita.

2. Age of the Lottery

A well-documented aspect of lottery products is that their growth is enormous at first, then tapers off and may actually decline. Initial growth occurs as potential players become

accustomed to the games and make ticket purchases part of their routines. No strong underlying theory predicts a later revenue decline, though two rationales present themselves. These are the wearing off of the lottery's novelty and the revision of expected utilities by consumers.

When a lottery is introduced into a state, it is quite possibly unlike any other product that state's consumers have had access to. A lottery could be the first form of gambling available to much of the law-abiding citizenry, or it could simply be the first accessible form of gambling--no travel is needed to a casino; no perceived skill is needed to handicap horses. Expectations that such novelty of a game will wear off are implicitly acknowledged by states' actions with lotto games--it is common that every few years the number of balls drawn or the number of drawings per week is changed to add new "excitement" to the game⁴. This acknowledgment is explicit as well--one lottery official remarked that "We compete for the entertainment dollar, and what may entertain people today may not entertain them tomorrow" (Arrarte, 1996).

Stover (1987) tests the idea that lotteries experience an initial burst in sales by including a dummy variable indicating whether the observation is the first year of a lottery. He finds first year sales to be substantially higher than sales in succeeding years because "[a] certain novelty effect initially causes many people to play the lottery; then a smaller but more constant following develops." An important note is that Stover only accounts for such an effect with the entire portfolio's *first* game. It is uncertain if such novelty induces greater purchases with every *game* introduced, with every *type of game* introduced, or solely with the *very first* lottery product. Also, he only allows for a novelty effect in the initial year. Though intuitively unlikely, the wearing-off process could conceivably take a longer amount of time.

A longer-term explanation for the aging process affecting sales is the revision of expected utilities by the game's potential players. Such an explanation does not follow directly from the model of the consumer presented above, but it does fit in nicely. Referring back to Figures 3 and 5, (pages 8 and 11) we see that the decision to play a lottery hinges upon the

expected utility exceeding the current utility. This expected utility, in turn, consists of knowledge of wealth changes and probabilities of these wealth changes. Kahneman and Tversky (1979) examine expected utilities in "Prospect Theory: An Analysis of Decisions Under Risk." For this study, their most important observation is that the choice process consists of "an early phase of editing . . . a preliminary analysis of the offered prospects, which often yields a simpler representation of those prospects." What guides this simplification process? In another article, Kahneman and Tversky (1982) explain that "a person could estimate . . . the likelihood of an event . . . by assessing the ease with which the relevant mental operation of retrieval, construction, or association can be carried out." Because many consumers consider lottery odds to be incomprehensible, it is accurate to say that such an estimation and simplification process occurs when they assess their chances of winning.

Lottery winners are well-publicized, especially the winners of large jackpots--their pictures appear in the paper and the evening news shows a jubilant family holding an oversized check with the winnings written upon it. By the arguments furnished by Kahneman and Tversky, people estimate their probability of winning by retrieving examples of winners versus examples of losers. Over time people may revise their expectations of winning. First, lottery winners receive less press as winners lose their novelty from the media's viewpoint. Scott Vandeman, who handles public information for the Florida Lottery, states that "where we [the Florida Lottery and its recent jackpot winner] once would get a 1/4-page feature with a picture, we now typically get three column inches in the 'Etc.' section with no photo" (Vandeman, 1997). Second, as time passes a person has a greater supply of personal experience with the lottery--and presumably of losing that lottery--to retrieve when estimating chances of winning. Former Vermont and West Virginia lottery director Ralph Peters remarks that "People . . . realize that not everyone will become a millionaire" (Knapp, 1988). Combined, these effects would cause the estimated probability of winning to decrease. Though the lottery game itself may not have changed, perceptions of probabilities--and

therefore expected utilities--would. If the expected utility decreased to a level lower than current utility the person would cease playing.

Table 2: Results from Prior Studies Regarding the Age of a Lottery Product

Study	Product	Relationship Modeled	Peak
Mikesell and Zorn (1987)	All	Quadratic	45 quarters
Mikesell (1987)	All	Quadratic	41 quarters
DeBoer (1990)	Lotto	Quadratic	25 months
Caudill, et al. (1995)	All	Linear	N/A

Table 2 summarizes results from papers that have tested lottery sales patterns over time. All studies that test for a quadratic relationship between time and sales discover a concave parabola. However, estimates for when a lottery's sales level peaks differ from about 2 years to over 11 years. These discrepancies most likely result from the lengthier estimates being based on all lottery games and the overall age, while the shorter estimate is based on solely the lotto game. For the entire lottery portfolio the revision of expectations could be offset or slowed by the introduction of new games, which may require a new process of estimation and revision. Caudill, et al. (1995) provide estimates for the age effect for samples of various time lengths, and finds that for lotteries that have been around longer (in their study, 10 years) the age has a greater positive effect. This appears to contradict the other studies, but in this case the dependent variable is net revenue rather than gross revenue. Conceivably, efficiencies in running the lottery could increase net revenue even if some sales decreases occurred.

We have then casual observations as well as empirical tests relating the age of a lottery product to increasing, then declining sales following an initial burst. Such a pattern has theoretical consistency. However, no determination has been made whether it exists for each product or if the entire portfolio's age dominates sales activity over time.

3. Substitution

States are in a virtual monopoly position with lottery products. Competition and potential substitution of products still exists, though, from three distinct sources. These are the following: other forms of gambling available to the state's residents, other lottery products offered by the state, and lottery products from neighboring states.

Common forms of legal gambling other than state lotteries are horse or dog racing and casino games. Lotteries and racing are fundamentally different--the acts of handicapping and choosing numbers for a random drawing are quite different types of consumption. Indeed, Clotfelter and Cook (1989) state that racetrack betting declines and lottery sales increases are not correlated, though their observation is not mathematically tested. In states with casino gambling, opportunities such as slots offer the same random, unskilled gambling that lotteries do. Clotfelter and Cook again remark that this form of gambling has not been affected by lottery adoptions, though once more no formal tests show this. Also, the causal relationship is stated backwards for this study's concern: lotteries may not affect casinos, but do casinos affect lotteries? In summary, though no effect has been found between other forms of gambling and lottery revenues, it has not been extensively analyzed.

Clotfelter and Cook (1990) argue that lotto sales growth does not harm the sales of other games within the state. They use two methods of cutting the numbers to come to this conclusion. First, the growth rates for other games decreased in only 4 of 13 states observed once lotto was introduced; the other 9 states witnessed increases. Also, in a week-by-week analysis, when lotto jackpots rolled over and caused enormous increases in sales, the sales for other games did not suffer a corresponding decrease. However, both Mikesell and Zorn (1987) and Stover (1990) find that sales of all games besides lotto suffer a tiny but statistically significant decrease when a lotto game has been introduced.

Regarding competition for consumers' lottery dollars between states, Stover (1990) again reinforces Mikesell and Zorn (1987). Both find that a state whose neighbors do not offer lottery products experiences higher sales of its own products. However, Vrooman

(1976) determines the presence of and expenditures on neighboring lotteries *increase* expenditures in a given state's lottery. This conclusion may reflect a population's willingness to bet--when people play more in one lottery, they are more likely to play in every lottery. His results do not necessarily contradict the first two studies.

Competition does appear to affect sales, but past studies have clearly determined neither the source nor direction of its effects.

4. Price

Since this study requires construction of a demand equation price ought to be included. Incorporating the price of a lottery ticket presents an interesting problem. First, it has been construed a number of ways: as the actual, "immediate" cost of a ticket (Scoggins, 1995); as the immediate cost divided by the game's expected value (Aronson et al., 1972); and, most often, as the effective price--the immediate price minus the game's expected value (Gulley and Scott, 1993; Clotfelter and Cook, 1993; Vrooman, 1976). Moreover, the expected value can be simply defined as the amount of the betting pool returned by the state to players in the form of prizes, or more exactly determined by using probabilistic models⁵.

Scoggins (1995) attacks the use of expected value in determining price, stating that "the explanatory variable known as the 'price' must not only be a dollar denominated variable but also must have a fixed relation to income *via* a budget constraint." I disagree with his argument because of the nature of the consumer. Previously it has been shown that the consumer's decision to purchase a lottery ticket rests upon the game's expected utility, which is correlated with its expected value. Given the minimal cost of each individual ticket it seems appropriate to consider price the same way a consumer does: as a current expenditure (which does not constitute a significant portion of the budget) combined with an anticipated payoff.

Obviously, as with any price, an increase (which would actually be a *decrease* in the expected value) should decrease the demand for the product. Both Gulley and Scott (1993) and Vrooman (1976) do find the expected signs on their price variables, though only the former elicits statistically significant results. Each study models price as the immediate price

minus the game's expected value, and, importantly, uses data for a shorter period than a year (Gulley and Scott use weekly data, while Vrooman uses monthly data). When data are measured annually the expected value does not change as drastically, but due to the basic theoretical importance of including a measure of a product's cost this study will include this measure of price.

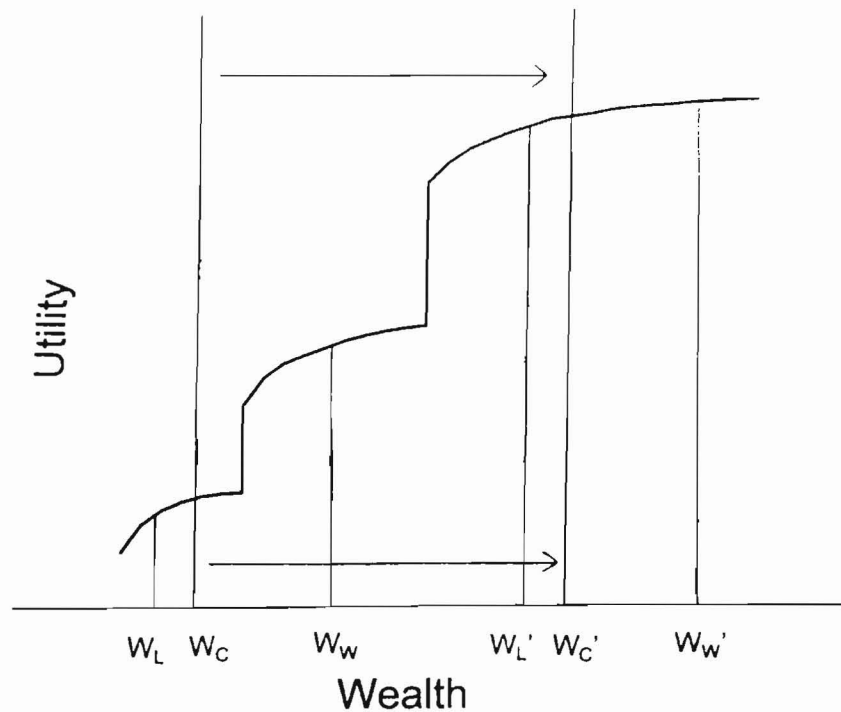
C. Demographic Variables

Along with those variables inherent in the lottery product, certain aspects of the player base affect sales levels. Past studies have examined a variety of such demographic variables. Levels of urban concentration (Stover, 1987; Clotfelter and Cook, 1993), religious concentration (Wu, 1979; Caudill et al., 1995) and unemployment (DeBoer, 1990; Mikesell and Zorn, 1987), as well as the player base's age distribution (Wu, 1979; Caudill et al., 1995) have shown either inconsistent or insignificant results and are thus ignored in this study due to the lack of theory requiring their inclusion. The racial breakdown of the state has been shown to have a consistent effect on sales, but not of lotto products. Clotfelter and Cook (1989) explain that race affects the numbers games, as they are "copies of illegal numbers games that . . . had thrived in minority neighborhoods for decades." A regression exclusive to lottos finds no significant impact on sales from race. Other variables, though--specifically the player base's wealth and education--do merit individual attention.

1. *Wealth of the Player Base*⁶

An integral part of the theory of the consumer and risk is the level of customary wealth. An increase in this wealth level would give the consumer a greater ability to purchase lotto tickets. A consumer actually purchasing more requires that the expected utility from playing remains above the current utility. Figure 7 demonstrates that this may not be the case. The figure shows an increase in customary wealth; the distances between W_L , W_C , and W_w are preserved with W_L' , W_C' , and W_w' --each is simply shifted to the right the same amount. This holds the changes in wealth offered by the lotto constant. As shown, when customary

Figure 7: A Change in Customary Wealth



wealth becomes W_C' the lotto's prizes may fall into one concave segment of the utility curve. This situation is identical to that modeled by the classical, strictly concave utility curve. Since no indivisibility sections are crossed, every possible expected utility lies below the current utility and the consumer would not play the lotto.

The utility curve presented has smaller wealth differentials between indivisibility points at lower wealth levels. Thus only at high wealth levels would the potential prizes and losses fall onto the same concave section. We have then a predicted quadratic relationship between the wealth of the player base and the sales of the lotto. Wealth increases cause sales increases up to a point, but when wealth surpasses some threshold level products will suffer a sales decrease.

All studies found by the author in which wealth partially explains the revenue generated by a lotto use annual income as a proxy for wealth. More importantly, they employ solely a linear income term, ignoring the quadratic one hypothesized here. Most find a positive relationship between average income and some form of sales--either sales, sales per capita, or

the logarithm of either of the previous two (see, for example, Mikesell and Zorn (1987), Mikesell (1987), and Caudill, et al. (1995)). In these studies the entire lottery portfolio is included in the same regression, so it is impossible to determine if income changes have different effects on various lottery instruments.

An early examination of the New York State Lottery (Vrooman, 1976) elicits an interesting result. It finds a significantly negative relationship between the total personal income and the sales revenue. This could be explained by the preceding theory if, for the observations, income is so high that further increases mean the negative effect of people not being enticed to play the lottery exceeds their ability to purchase more tickets. Since Vrooman only includes a linear term this would result in the negative coefficient found. Thus some weak evidence exists that implies a quadratic relationship between customary wealth and sales.

Further evidence can be found in an analysis of demographic studies conducted by various states. Table 3 presents the results of looking at the average income levels for players of lotto-style games. In general, as lotto games have more balls the jackpot increases. A correlation is apparent, then, between the income level of the players and the possible jackpot. This means that higher-income individuals do not play games with lower jackpots as much as they play those with higher jackpots. Unfortunately, due to small sample sizes, the statistical validity of this relationship is quite weak.

Table 3: Average Income of Lotto Game Players, as a Ratio of State Average Income

Game	Number of Observations	Average Income	t-test
5-Ball	7	.98	between 5 and 6 = 1.50*
6-Ball	9	1.05	between 6 and Multi = 1.24
Multi-state	4	1.12	between 5 and Multi = 2.32**

* = significant at the 0.10 level

** = significant at the 0.05 level

*** = significant at the 0.01 level

2. Education Level of the Player Base

Clotfelter and Cook (1989) forcefully state that "there is no more clear-cut correlation with lottery participation: lottery play falls with formal education." Examining their regression, though, reveals that this relationship only holds for the *amount wagered* by players. In the model determining *whether a consumer plays* the lottery, the relationship with education is less consistent. However, Wu (1979) and Jackson (1994) both find a negative relationship between education and demand. Wu discovers that increased education negatively influences the decision to purchase tickets as well as the amount bought, given that a person does purchase tickets. Jackson's analysis shows that increased education adversely affects sales both overall and for individual games.

As with the age of the lottery product, there is no immediately evident shift in the consumer model presented resulting from a change in education. However, an increase in education could affect consumers' interpretations of probabilities. If a more educated person were more apt to revise the probability of winning downward, that person's expected utility would be more likely to fall below his current utility. As stated numerous times before, when the expected utility is less than the current utility, the consumer will not purchase lottery tickets for the given game.

D. Summary

Before constructing the equation to examine demand for lotto a review is beneficial. First, Figure 5 (on page 11) summarizes a combination of utility theories that creates a model of consumers' behavior under risk. This model is used to examine specific variables' effects on lottery sales from a theoretical perspective. Other variables are included based on their importance as determined by prior empirical studies. The first variables considered are those aspects of the lotto game that affect demand. These are the potential prizes, the age of the game or of the entire portfolio, substitution from similar products, and the price of the tickets.

The second factors that affect the demand are rooted in the potential players' characteristics. For lotto products, the player base's income and education levels are relevant.

IV. EMPIRICAL MODEL

The preceding section examined and justified certain variables' inclusion in the demand equation for lotto sales. In general, the function predicted is:

$$D = f(Z, A, O, P, W, E)$$

where D = Demand, Z = Prizes, A = Age of the lotto, O = Other gambling opportunities available, P = Price of the tickets, W = Wealth of the player base, and E = Education of the player base.

There are a number of specific hypotheses related to what will influence the sales of a multi-state lotto. Since this study aims to identify and quantify the rationale for multi-state lotto games, particular attention should be paid to hypotheses related to lotto changes rather than those determined by demographic characteristics of the population--which are not affected by introducing a multi-state lotto. These "most important" hypotheses this paper tests are the following:

- 1) An increase in the prizes will increase the demand for a lotto game.
- 2) As a lotto product ages it elicits increasing demand, which peaks and then begins to decline.
- 3) The existence of other lotto products in both the state and the states that border it affect demand. Though past studies do not agree on the direction of this effect, it is more intuitive to assume a negative one.
- 4) An increased price for the lotto will have a negative effect on demand.

Those hypotheses relating the demographics of the state to the demand for a lotto product are the following:

- 1) As the income of the state's residents increases a demand peak is reached, followed by a decrease in demand with further increases in income.

- 2) As the education level of the state's residents increases, the demand for the lotto product decreases.

The regression includes only pari-mutuel lotto games--lottery products that require a player to choose a subset of numbers from a set of balls and match all of the ones the state draws in order to win the jackpot prize. Revenue per household measures demand. Observations come from 21 different states⁷, over the years 1984 to 1994 inclusive, with each observation consisting of a fiscal year for a lotto product in a state. This means that multi-state lottos have multiple data observations--each state reports its revenue separately. Using different states in the same regression is justified because, as mentioned before, states have long offered similar portfolios to their respective populations. Because different states are used, though, different sets of consumers determine the lotto products' demand. An implicit assumption made in this analysis is that regions as large as states have similar aggregate consumer demand patterns regarding a lotto, holding the included variables constant. On another technical note, all dollar amounts are adjusted to be constant 1984 dollars; the conversion factor is the arithmetic average of the monthly CPI for the appropriate fiscal year.

Regression results were generated using the ySTAT software package, and initial regressions are OLS. No functional transformations are used in the regressions (e.g. using log-linear forms) because there exists no *a priori* reason for doing so. The initial regression to test the hypotheses is shown below; refer to Table 4 for formal variable definitions and sources.

$$\begin{aligned} \text{REVENUE_PERHH}_t = & B_1 + B_2\text{JACKPOT}_t + B_3\text{AGE}_t + B_4\text{AGE_SQ}_t + \\ & B_5\text{ONLY}_t + B_6\text{BORDER \%}_t + B_7\text{EXPECTED LOSS}_t + B_8\text{INCOME}_t + \\ & B_9\text{INCOME_SQ}_t + B_{10}\text{EDUCATION}_t + B_{11}\text{MULTI}_t + u_t \end{aligned}$$

Revenue is compared across states and across time, so it must be per capitized by some method. This study uses a state's household population rather than the population count of individuals for two reasons. First, the necessary age for legally purchasing a lotto ticket (usually 18 or older) means that much of the population is ineligible to exercise its demand by

Table 4: Variable Definitions and Sources

Variable	Exp. Sign	Definition
REVENUE PERHH		Revenue per household of the Lotto for the fiscal year, in dollars. Revenue numbers are from <i>The '96 World Lottery Almanac</i> . The number of households comes from <i>Current Population Reports</i> , series P-25.
JACKPOT	+	The average jackpot value for the year, in millions of dollars. Unpublished data from correspondence with state lottery bureaus.
AGE	+	The game, lotto, or lottery age in years, measured at the fiscal year's midpoint.
AGE_SQ	-	The age squared.
ONLY	+	1 if the game is the only lotto-type product in the state for at least half the fiscal year. 0 otherwise.
BORDER %	-	The number of border states offering lotto games for at least half the given fiscal year divided by the number of border states, times 100.
EXPECTED LOSS	-	The expected loss of the lotto as a percentage, equal to $(1 - \text{payout rate}) \times 100$. The payout rate is the state expenditure on prizes for the game divided by the game's revenue.
INCOME	+	The median money income of households, from <i>Current Population Reports</i> , series P-60.
INCOME_SQ	-	Median money income of households squared.
EDUCATION	-	The percentage of the state's residents who are at least 25 years old and have completed high school. From <i>Current Population Reports</i> , series P-20.
MULTI	?	1 if the lotto product is a multi-state one. 0 otherwise.

contributing to revenue. Second, the author feels that the purchase of lotto tickets is not, most accurately, an individual's action. The decision to purchase a lotto ticket as well as any associated winnings are shared on the household level. If the husband purchases a ticket, the wife's demand for playing a lotto is conceivably satisfied; the household count is more appropriate in adjusting revenue numbers.

JACKPOT measures prizes for this study. Though multiple tiers of prizes exist, studies examining prize distribution measures have not elicited meaningful results, as described in section III. However, the jackpot level has shown a consistently dominant effect on the level of sales. Moreover, the price variable (EXPECTED LOSS) takes into account the overall prize structure. The option of using guaranteed *minimum* jackpots exists, but the average offered jackpot over the course of the fiscal year is a better measure. The average measure will reflect any extraordinary jackpot levels reached due to successive rollovers, which would have caused a sales frenzy and higher revenue amount.

A potential difficulty in using JACKPOT as an independent variable is the fact that the pari-mutuel jackpot, by definition, depends on the sales level--the independent variable depends on the variable it is explaining. The author feels that this situation should not invalidate results because the jackpot level is announced prior to the sales generated by it. In other words, sales levels affect the jackpot of subsequent time periods, and then only if a rollover occurs. Also, in practice the jackpot level acts as an independent variable causing sales--states announce the next drawing's jackpot before any sales have occurred.

AGE and AGE_SQ together test hypothesis 2. A positive coefficient on the linear term and negative one on the quadratic term will result in the hypothesized increasing, peaking, and decreasing revenue pattern. By ignoring the first partial year of sales for a lotto, the novelty effect found by Stover (1987) should be avoided, and only the more steady aging process should be evident. As mentioned in section III, theory does not specify which measure of age determines such consumer revisions. Three measures are plausible: the age of the *game*, the age of all *lotto* products in the state, and the age of the entire *lottery* in the state. For example, assume a state first offered a lottery product (say, instant tickets) in 1980, offered its first lotto product in 1985, and initiated another lotto product in 1990. The three age measures for the newest lotto product in 1992 would be 2, 7, and 12. Each measure will be tested in order to determine which fits the data most accurately.

ONLY and BORDER % test the third hypothesis by including substitution effects. ONLY accounts for multiple lotto products *within* the state competing for a consumer's lotto dollar. BORDER %, on the other hand, accounts for interstate competition. Though an imperfect measure, BORDER % provides more accuracy than a simple dummy variable regarding the presence of interstate competition. Stover (1990) models interstate substitution much more exactly by examining population counts in each neighbor state's bordering counties. Since substitution is not this study's sole focus such detail is unnecessary. Also, no measure for substitution from other forms of gambling exists in this study.

EXPECTED LOSS tests the final lotto-determined hypothesis. As described in section III some debate exists regarding how to measure the price of a lotto ticket, though most researchers use the payout percentage. That measure is therefore used here as well. The expected loss, equal to 1 minus the payout rate, is used instead of the payout rate itself simply to measure the price in such a way that a negative sign is expected. As mentioned above, EXPECTED LOSS and JACKPOT together portray the overall prize structure.

INCOME and INCOME_SQ formally test whether consumer income changes are quadratically related to the lotto's revenue. As with the AGE variable, a positive linear and negative quadratic term will generate a curve with the theorized shape. Also, as with the revenue numbers, the measurement of INCOME is on a household basis.

EDUCATION is a simple measure of the population's education. A breakdown of education levels would present more detail, but given the focus of this study it would be redundant. Due to sporadically reported measurements of this variable the author used linear interpolation in order to have successive years of observation⁸. This obviously causes some inaccuracy problems, but they are not egregious. Education levels in a given state do not change drastically in the space of one or two years. Rather, it is the difference in education levels *between* states that generates the variation of interest for this regression.

The final variable included is MULTI, which indicates whether a game is a multi-state one or not. This is important due to the assumption that a multi-state lotto differs from a

single-state one, from the consumer's point of view, only in the variables included: its jackpot, its age, relevant substitution effects, and its overall prize structure (price). If this assumption is indeed correct, MULTI should have a coefficient statistically indistinguishable from zero. A potential shortcoming of this study is that, although observations of multi-state lottos come from many different states, all of them are for Powerball⁸. Therefore, the results will attribute any effects inherent in Powerball to the product being a multi-state one.

V. RESULTS

Results are divided into two sections. The first describes regression results and hypothesis testing, while the second uses the regressions to make predictions for The Big Game as well as Illinois' other lotto products.

A. Regression Results

Having tested each possible age measurement alone and in combination with the others, the author found the age of the entire lottery portfolio to best describe revenue. Therefore this measure of age is used throughout the regressions. Table 5 presents results for all models run.

In the originally formulated model two statistical problems are apparent. First, the Durbin-Watson statistic of 0.828 means that having no serial correlation is rejected at the $\alpha = 1\%$ significance level. For Model 1 the data were ordered by state name (alphabetically), then by game name, then by year. The final sorting order of year means this cross-sectional analysis acts as a time series over short stretches of observations. A missing variable exhibiting a cyclical pattern could lead to the serial correlation of the errors. Another potential source is the mis-specification of a functional form for a variable. Examining graphs of the error terms against each independent variable showed the errors have a strong correlation with JACKPOT. This leads to the second, and related, problem. Model 1 is severely heteroskedastic with relation to JACKPOT. The Goldfeld-Quandt test (Ramanathan, 1995) rejects having error terms with constant variance at the $\alpha = 1\%$ level. The jackpot

level has a large and highly significant effect on revenue; such an important variable quite reasonably would result in more variable revenue.

Table 5: Results
Dependent Variable = REVENUE PERHH

Variable	Exp. Sign	Model 1 Coefficient (t-statistic)	Model 2 Coefficient (t-statistic)	Model 3 Coefficient (t-statistic)	Model 4 Coefficient (t-statistic)	Model 5 Coefficient (t-statistic)
JACKPOT	+	3.424*** (7.755)	11.480*** (8.960)	11.337*** (9.299)	15.391*** (5.773)	14.916*** (7.725)
JACKPOT_SQ	?		-0.285*** (-6.586)	-0.280*** (-6.799)	-0.691*** (-2.831)	-0.771*** (-4.166)
JACKPOT_CU	?				0.011* (1.706)	0.014*** (2.808)
AGE	+	0.333 (0.208)	2.789** (1.951)	2.778** (1.951)	2.858** (2.022)	2.658*** (2.923)
AGE_SQ	-	-0.011 (-0.147)	-0.169*** (-2.393)	-0.168*** (-2.393)	-0.175*** (-2.506)	-0.160*** (-3.433)
ONLY	+	7.365* (1.393)	4.961 (1.083)	5.114 (1.125)	5.483 (1.214)	10.053*** (3.666)
BORDER %	-	-0.341*** (-4.288)	-0.275*** (-3.958)	-0.271*** (-3.958)	-0.244*** (-3.504)	-0.175*** (-3.715)
EXPECTED LOSS	-	0.866* (1.895)	1.245*** (3.121)	1.229*** (3.110)	1.345*** (3.378)	1.295*** (4.597)
INCOME	+	12.972** (1.889)	2.512 (0.409)	4.790*** (5.392)	4.721*** (5.351)	3.977*** (6.168)
INCOME_SQ	-	-0.210* (-1.410)	0.050 (0.375)			
EDUCATION	-	-2.619*** (-5.758)	-2.196*** (-5.517)	-2.258*** (-6.269)	-2.226*** (-6.219)	-1.945*** (-8.187)
MULTI	?	-55.409*** (-8.604)	-71.134*** (-11.756)	-70.798*** (-11.873)	-70.650*** (-11.938)	-64.673*** (-15.665)
CONSTANT		48.244	54.959	35.877	18.198	13.673
Adjusted R ²		0.641	0.733	0.735	0.739	Invalid
Durbin-Watson		0.828	1.824	1.836	1.852	1.836

* = significant at the 0.10 level

N = 135

** = significant at the 0.05 level

*** = significant at the 0.01 level

Introducing a quadratic term for JACKPOT attempts to correct for the statistical problems. The results are shown as Model 2. The adjusted R^2 improves noticeably, as does the Durbin-Watson statistic¹⁰. Coefficients are stable, except for those on INCOME and INCOME_SQ. Comparing Models 1 and 2 we see that the effect of income on revenue per household changes its functional shape and also loses its significance. Both coefficients being positive implies that the effect of the player base's income on revenue may more accurately be an upward-sloping line. Model 3 is identical to Model 2, except that income is constrained to a linear effect.

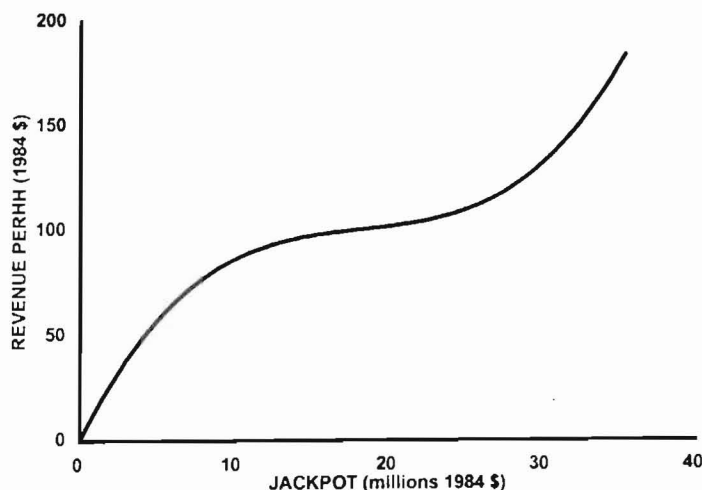
Model 3, then, performs consistently with Model 2, with the linear income term having a strong positive effect on revenue. Before continuing, note that ONLY, though not statistically significant at normal levels, has been retained throughout the different models. This is because ONLY has a stable effect on revenue, and its counterpart variable measuring competition among lotto products (BORDER %) has a similarly stable and significant effect. Together, these facts imply that keeping ONLY in the model adds to its predictive powers.

A subsequent test of Model 3 to check if the heteroskedasticity had been corrected failed--the size of the jackpot still resulted in non-constant variance in the error terms. The mechanics of running the Goldfeld-Quandt test on Model 3 suggested that a cubic JACKPOT term may more accurately predict the revenue generated by a lotto product¹¹. Such a form is allowed for in Model 4. After Model 4, having explored two functional changes in JACKPOT and still finding heteroskedasticity, the author elected to use a weighted least squares correction (Ramanathan, 1995) rather than explore yet more exotic functional possibilities. Model 4 results are consistent with those of previous models; moreover, the cubic JACKPOT term brings additional explanatory power to the regression. Thus the weighted least squares regression, shown as Model 5, retains the variables of Model 4. Specific hypotheses are examined with the coefficients from this last model.

The first hypothesis--that an increase in prizes will increase the revenue of the lotto--has the most complex answer of this study. The hypothesis is confirmed, since the coefficients

indicate a constantly increasing function. Graphically, Figure 8 presents the relationship of the jackpot level and the revenue generated per household. Mathematically, $\partial R / \partial J > 0$ for every J (where R = REVENUE PERHH and J = JACKPOT). However, the theory gave no indication that a cubic relationship would exist between the two variables. A possible explanation for it is as follows¹². Many successive rollovers--many drawings without a winner--cause larger jackpots. A well-known fact of lotto products is that if nobody wins then potential players lose interest in the game. This follows from the argument presented earlier that potential players retrieve examples of winners in order to estimate their possibilities of winning. When nobody wins, expectations of winning are revised downward, and demand for the lotto is reduced. This could explain the early part of the jackpot / revenue curve. Though an increase in the jackpot level appeals to potential players through a larger possible prize, some potential players lose interest in the lotto and offset the larger-prize effect.

FIGURE 8: Relationship of JACKPOT to REVENUE PERHH



By the regression results, this relationship holds for an average weekly jackpot up to \$18.36 million. Past that level, further increases in the jackpot cause accelerating demand--demand increases at an increasing rate. This could be due to enormous jackpots overcoming the effects of potential players deciding they have too low a probability of winning. In a sense, the effect of successive rollovers may have already taken its toll on changing players' estimates of winning; further jackpot increases have no adverse effect. In this case only the originally-hypothesized relationship of a higher jackpot increasing demand remains.

In terms of magnitudes the jackpot has a sizable effect away from the inflection point. For example, in Illinois the average weekly jackpot in 1990 was \$10.024 million, while in 1991 it was \$14.686 million. Applying the coefficients, this change between years in the average weekly jackpot level would result in an increase of revenue per household of nearly \$11.

The model supports the hypothesis of a pattern of increasing, peaking, then decreasing revenue as the lotto ages--each of the linear and quadratic formulations of age is significant at the 0.01 level. Their combined effect results in a peak age of a little over 8 years. This fits in well with the peaks found by Mikesell and Zorn (1987) and Mikesell (1987) of 45 quarters and 41 quarters, respectively.

A more interesting aspect of the age variable is that the entire portfolio age fits the data closer than the game age or lotto age. This implies two things. First, the act of revising expectations concerning potential winnings occurs with regard to all lottery products, not separately for each one. This makes sense in terms of the theory presented, where media interest dies out and examples of personal losses accumulate. Such effects would indeed occur with regard to the entire portfolio of products. A second implication is that a new game, such as a new multi-state lotto, is *not* seen by the potential players as a product offering new opportunities. The novelty effect may still hold (by ignoring the first year of sales this was not tested), but overall the state's history of offering lottery products dominates the aging effect of the "new" game.

This study contributes to the debate regarding the effects of competition between lotto products. If a lotto is the only one in the state, then the additional revenue it generates is about \$10 per household, in 1984 dollars. This effect is more accurately interpreted as: a lotto will generate \$10 per household *less* when another lotto product is introduced to the state, *ceteris paribus*. Also, lotto products offered in bordering states cause revenue decreases for each other. The magnitude of BORDER % appears tiny when compared to that of ONLY, but its effect can be greater. If, for a given state, none of the bordering states offered a lotto product (BORDER % = 0), then the effect of interstate competition on revenue would be \$0. If instead all of the bordering states offered lotto products (BORDER % = 100), then revenue would be \$17.50 lower per household--by no means a tiny effect.

The fourth hypothesis--that an increased price will cause a decrease in revenue--is firmly rejected. Throughout the various formulations of the model EXPECTED LOSS has a significant and positive effect. The Model 5 coefficient says that if the expected loss of a lotto increases from 45% to 50% then the revenue generated by that lotto will *increase* about \$6.50 per household. Such an upward-sloping demand curve is indeed puzzling. A possible explanation is that when considering the purchase of a lotto product consumers focus solely on the jackpot, ignoring the overall prize distribution and resulting expected value. Potential players would then be satisfied as long as enough of the betting pool goes to prizes to generate reasonable jackpots. This explanation is plausible given the difficulty of determining expected winnings in lottos, but it implies only an insignificant coefficient--not a strongly positive one. Moreover, the positive result contradicts previous findings where higher prices negatively affect demand.

The exclusion of a variable specifically measuring jackpot levels in the studies finding the predicted price / demand relationship suggests a rationale for this study's contradictory finding. Both Vrooman (1976) and Gulley and Scott (1993) examine *one* lotto product with observations occurring at short time periods--monthly and weekly, respectively. Whereas this study uses an overall expected loss and considers jackpot levels independently, the others

include varying jackpot levels only to the extent that they alter an individual drawing's expected value. They find that an increased expected value (decreased price) causes higher sales. Since this increased expected value does not indicate a fundamental change in the game, but rather a change in the jackpot, perhaps their results are more appropriately comparable to the positive JACKPOT coefficient in this study. Again, however, such an explanation does not account for this study's positive and significant coefficient as opposed to one equaling zero. This issue requires further exploration.

Compared to the results for lotto-related hypotheses, the results for demographics-related hypotheses are straightforward. Model 1 appears to validate the original concept that the potential players' income level has a quadratic relationship with demand. Due to the sensitivity of this result to model specification, this hypothesis ended up not being confirmed. However, results are firmly in line with previous studies that have found a highly significant, positive effect of income increases on demand for lottos. This study determines that an increase in median household income of \$1000 results in an increase of about \$4 of revenue per household. Finding that a linear relationship fits the data better than a quadratic one does not necessarily refute the theory presented. This simply means that over the median income levels examined in this data set, which range from \$14,468 to \$30,894, the increased budget effect greatly exceeds the lack of interest effect. In this income range people do not accumulate enough wealth to cause a lotto's potential winnings to remain in the same concave portion of the utility curve as the customary wealth, as was shown in Figure 7 (page 21). Given the multi-million dollar prizes possible and comparatively low incomes, this is reasonable. A study testing the quadratic relationship and using smaller data units--for example, individual cities or families--would allow greater variation in incomes. This would possibly provide examples of those with wealth levels high enough to be past the threshold point of having interest in lottos.

Finally, EDUCATION performs as expected--more educated populations, *ceteris paribus*, exhibit less demand for lottos. A lotto generates about \$2 less revenue per household for each percentage point increase in the population having at least finished high school.

In summary, this study presents a rather bleak picture for lottos. A newly-introduced lotto, despite being a different game, is subject to dwindling demand caused by the state's overall portfolio age. Moreover, the game detracts from the revenue of the other lotto products within the state by acting as a substitute. It is left up to the jackpot, then, to allow the new game to generate more revenue than its alternatives. This is why the multi-state lotto is the necessary next step in a lotto portfolio. Such games have the population base required to generate revenues that allow otherwise-unattainable jackpot levels. Indeed, these jackpots are designed to be monstrous. For example, The Big Game has offered a jackpot of \$77 million within its first 7 months; never in the 14-year history of the Illinois Lotto has its jackpot reached this level.

The performance of MULTI casts a grim shadow over the notion of a multi-state lotto reviving revenues. By Model 5, if a lotto is a multi-state one it suffers a decrease in revenue of almost \$65 per household. This implies that the variables included in the regression portraying a lotto product do not wholly do so; a multi-state lotto differs from a single-state one in some way not measured. As mentioned earlier, sampling limitations could mean this is actually a Powerball effect that does not hold for every multi-state lotto. Also, multi-state lottos differ from single-state lottos most noticeably in the JACKPOT measurement--in this study's sample the average value of JACKPOT for single-state games is 5.95, while for multi-state games it is 14.81. Since the problem of formulating the jackpot level was never fully solved, further explorations of how the jackpot affects revenue may prove this multi-state effect spurious.

However, the possibility obviously remains that a multi-state lotto simply generates less revenue than an otherwise-identical single-state one. This is seen in the observations used for the regression, where the average value of REVENUE PERHH is 69.56 for single-state lottos

but only 30.63 for multi-state ones. A potential reason for this was thought to be the fact that the multi-state observations represent lottos that are *not* the first ones in the state. Many players could be "faithful" to the lotto they already play and thus not participate in the newer multi-state product. Including a dummy variable indicating whether a lotto was the first such product in the state resulted in an insignificant coefficient. Moreover, the large and negative MULTI coefficient remained.

Another possible explanation for this is the nature of potential consumers estimating their probabilities of winning. They may consider the vaster number of people playing the same lotto and rationalize that the chance of being the sole winner is greatly reduced. This is, in fact, the case--and in the estimation process, thinking of many states' residents as "competitors" for the prize may mean the difference between thinking one could win and one could not.

B. Predictions for Illinois' Lotto Products

Using the coefficient predictions of Model 5 this study now makes predictions for Illinois' lotto products--Little Lotto, Lotto, and The Big Game. First, *ex post* forecasts check the performance of the results. For the regressions run above Illinois observations were excluded; Table 6 presents their *ex post* forecasts.

This table illustrates a few points. First, the model predicts much more accurately for the Lotto game from a percentage error viewpoint. This is expected because the Little Lotto jackpot amounts are some of the lowest in the regression, while the Lotto jackpots are closer to the mean jackpot amount. Second, the sales from Lotto have experienced a steady decline since 1987; the regression predicts this decline as of 1990, and models it fairly well. Finally, the model consistently underestimates for Illinois, with a bias of a bit over \$10.50 per household for each lotto offered. This could reflect some overall gambling propensity of Illinois citizens, which would appear to be a type of regional effect. But, as mentioned in part

A of the results section, regional dummies did not enhance the model. Still, the underprediction should be considered when examining the *ex ante* forecasts.

Table 6: *Ex Post* Forecasts of Illinois Lotto Revenues
(all revenue amounts in constant 1984 dollars per household)

Game	Fiscal Year	Revenue	Predicted Revenue	Absolute Difference	Percentage Difference
Little Lotto	1989	\$31.83	\$38.27	\$6.44	20.24%
	1990	26.98	28.55	1.56	5.79
	1991	21.76	7.29	-14.47	-66.50
	1992	20.78	2.62	-18.16	-87.40
	1993	21.22	0.56	-20.66	-97.36
	1994	19.5	0.38	-19.12	-98.06
Average, Little Lotto		23.68	12.95	-10.74	-53.88
Lotto	1985	127.59	125.16	-2.43	-1.91
	1986	139.41	115.2	-24.22	-17.37
	1987	143.01	108.72	-34.29	-23.98
	1988	120.10	97.09	-23.01	-19.16
	1989	117.74	116.29	-1.44	-1.23
	1990	110.53	101.86	-8.67	-7.85
	1991	105.97	95.59	-10.39	-9.80
	1992	106.94	90.01	-16.93	-15.83
	1993	80.02	83.74	3.72	4.65
	1994	64.04	75.98	11.94	18.64
Average, Lotto		111.54	100.96	-10.57	-7.38

Table 7 presents the *ex ante* forecasts. The following variables required estimation: JACKPOT, INCOME, EXPECTED LOSS, and EDUCATION. EXPECTED LOSS is the average of the past expected losses for Little Lotto and Lotto, while equal to 50 for The Big Game as stated in its official rules. For the other three variables, the following two methods

of estimation were used: linear regression based on the last four observations (Table 7A) and the arithmetic mean based on the last four observations (Table 7B). These two methods apply for each variable except in the case of JACKPOT for The Big Game. Since there is no four-year history to this game both predictions use the average of all its offered jackpots from its inception to March 28, 1997.

Table 7A: *Ex Ante* Forecasts of Illinois Lotto Revenues, Linear Regression
(all revenue amounts in constant 1984 dollars per household)

Fiscal Year	Little Lotto Prediction	Lotto Prediction	The Big Game Prediction
1998	-\$18.02	\$57.76	\$8.22
1999	-23.09	49.57	4.39
2000	-28.49	40.42	0.23
2001	-33.24	30.26	-4.25
2002	-38.04	19.02	-9.05

Table 7B: *Ex Ante* Forecasts of Illinois Lotto Revenues, Constant Mean
(all revenue amounts in constant 1984 dollars per household)

Fiscal Year	Little Lotto Prediction	Lotto Prediction	The Big Game Prediction
1998	-\$20.18	\$62.27	\$1.84
1999	-25.18	57.27	-3.16
2000	-30.49	51.96	-8.48
2001	-36.13	46.32	-14.11
2002	-42.09	40.36	-20.07

At first these predictions appear dubious because a number of them are negative, but upon further review they may not be. Illinois is currently in a terrible situation from the perspective of this study's revenue estimation. First, it obviously offers more than one lotto product--so ONLY decreases REVENUE PERHH by more than \$10. Also, every bordering state has offered at least one lotto product since 1990, resulting in an estimated demand decrease of over \$17 per household.

The advanced age of the Illinois Lottery causes both appropriate decreases in predicted revenues and an imprecise decrease based on statistical problems. In the regression sample the average age is 8.0 and the largest is 20.6. However, for the Illinois predictions the age ranges from 23.4 to 27.4 years. Such age values exacerbate the downward side of the quadratic effect found (between 2001 and 2002 the extra year alone causes a predicted revenue decrease of almost \$6). Also, predictions based on variable values outside the range comprising the sample data suffer greater inaccuracy.

Generalities evident in the predictions do offer some insight, though. First, Little Lotto does not appear to be a viable game even in the near future. Its jackpot has been steadily decreasing in real value over its entire life, and sales have similarly suffered as shown in the "Revenue" column of Table 6. Though its revenue will never be negative, the Lottery Bureau may end Little Lotto due to its unimpressive contributions to the portfolio. In essence, its "little" jackpots do not begin to overcome the effects of the aging lottery portfolio and substitution from other games. Lotto, though having positive predicted revenues, does not fare any better--like Little Lotto, its jackpots and revenues have been decreasing steadily of late, and nothing would indicate that a reversal of this trend should occur. As with all the games, the effect of the age may not be as severe as presented, but it should still decrease revenues as the years pass.

Finally, The Big Game's predicted revenues are greatly discouraging from the Lottery Bureau's perspective. The two main differences for this new multi-state game were to be its increased jackpots and "newness" to consumers. This study has found that such newness does not actually occur in regards to generating rising sales. Thus its jackpots represent the other opportunity for The Big Game to distinguish itself. So far, though, the average weekly jackpot has only been \$20.193 million--\$12.732 million in 1984 terms. This is *not* past the estimated inflection point on the jackpot / revenue curve, so the area of accelerating demand is not utilized to drive sales. The differential in estimated jackpots does give The Big Game about a \$23 per household advantage over Lotto, but this advantage is too small to overcome

the effect of it being a multi-state game. Even if this multi-state effect of a \$65 revenue decrease is spurious, The Big Game would not provide sales levels reached by Lotto in its heyday of the mid-1980s.

In summary, The Big Game does not appear to be big enough to bring in consumers' dollars, and therefore its revenues will not be satisfactory to the Illinois Lottery Bureau. Indeed, Illinois officials already have expressed discontent with the multi-state lotto. A recent newspaper article explained that "Illinois is the biggest state offering The Big Game, but ticket sales lag behind those in Georgia" (Novak, 1997). Note that Georgia, with its much lower lottery age (the first game was offered in June of 1993) has seen greater revenues than Illinois from The Big Game.

VI. CONCLUSION

In response to the recent growth of multi-state lotto games, this paper seeks to understand the economic rationale of such products. The analysis first examines alternatives to the classical depiction of a consumer's total utility curve being always differentiable and strictly concave. A hybrid model is developed, incorporating concepts from Friedman and Savage (1948), Markowitz (1952), and Kwang (1965). A number of hypotheses relating changes in a lotto product or the demographics of a lotto's potential players are tested in order to determine what a multi-state lotto product offers as opposed to a new single-state product.

For the most part, results from the model agree with those elicited by previous studies of lotto products. The average weekly jackpot offered has a complex but enormous effect on the demand for a lotto; this requires additional exploration if state lottery bureaus wish to use the jackpot-producing capabilities of multi-state products effectively. For example, if the cubic relationship accurately portrays the relationship between jackpots and revenue, lottery officials should be sure to design games with average jackpots exceeding the inflection point of around \$18 million per week--\$28.5 million in today's dollars.

Along with investigating exactly how the jackpot affects demand, three areas deserving future consideration have already been mentioned in the results section. First is the theorized but unsuccessfully validated hypothesis that potential player income increases will eventually decrease the revenue, which requires a smaller observation unit for proper testing. Others are the effect of the overall prize structure (i.e. the expected loss) on demand and whether all multi-state lotto products suffer a large decrease in demand.

As with any research, this project uncovers more questions than it answers. Despite these new concerns, though, it offers some insight regarding multi-state lotto products. In summary, the jackpot is the key to their success. The state offering the product may find the revenues from the product disappointing because of the advanced age of the lottery portfolio and the effects of having competing products within the state. Previously unheard-of jackpots must drive demand by sparking enthusiasm in the potential players. As more states offer lotto products, specifically multi-state lottos, competition will increase between games and growth opportunities will exhaust themselves. Even currently, jackpots in multi-state lottos may be inadequate to fuel satisfactory revenues.

In Illinois' case, The Big Game enters an aging lottery portfolio, one with two older lotto games that have seen drastic declines in revenue over the past few years. The predictions show that these declines will continue for both older products. Additionally, The Big Game does not seem to offer jackpots that will return revenues to levels seen earlier in the Illinois Lottery's history. Illinois appears to have two options. First, it can explore streamlining its lotto offerings to minimize substitution effects and then look into a mega-multi-state lotto, one with jackpots that regularly exceed the inflection point of 28.5 million current dollars. Second, it can focus on non-lotto products--instant games continue to do well despite their long history, and Video Lottery Terminals are a recent success in many states. In any case, this study finds that without changes in the lotto environment or prize stipulations, Illinois will not receive desired revenues from its multi-state lotto.

FOOTNOTES

1. In reality there may be a limited supply of tickets. For example, Clotfelter and Cook (1989) report that sales for a single number combination in the Massachusetts Daily Game are capped at \$5 million. Since the prizes in the daily game are not pari-mutuel, the state could suffer large losses if a popular number, such as 333, were drawn.
2. Lotteries are "unfair" games; i.e. they have negative expected values from the viewpoint of a player. A portion of the wagers (usually around 50%) is used to cover administrative costs and is contributed to the state fund. Since not all wagers are returned to the players as prizes the expected value is negative.
3. A numerical example would clarify this point. Say one indivisible good occurs at \$50,000 and the next at \$200,000. This is a \$150,000 difference arithmetically, while a 4:1 difference in terms of ratios. Say another indivisible good occurs at \$1,000,000. This study assumes that it makes more sense to consider the next one being at around \$4,000,000 (a 4:1 ratio) than \$1,150,000. The latter would preserve the \$150,000 arithmetic difference but reduce the ratio to 1.15:1.
4. Below is a list of dates and corresponding changes to the Illinois Lotto. Though no structural changes have occurred since August of 1990, the starting jackpot has been altered a few times. Notably, in September of 1993 the starting jackpot dropped to \$2 million. Also, since The Big Game's inception in August of 1996 the starting jackpot has remained at \$2 million, but a rollover results in a lower addition ("Lotto," 1996).

Date	Change	Example of Starting Jackpot
2/19/83	Start	\$1 million
5/19/84	To 6/44 matrix from 6/40 + 1/40	\$6 million
1/11/86	To 2 draws per week from 1	\$2 million Wed., \$5 million Sat.
4/18/87	To 1 draw per week	\$4 million
5/7/88	To 6/54 matrix	\$5 million
8/11/90	To 2 draws per week	\$3 million

5. The conceptual formula is $EV = (\text{probability of win}) * (\text{jackpot}) * (\text{expected share of jackpot})$. Refer to Clotfelter and Cook (1991) for a detailed explanation of this.
6. Many studies examining the issue of wealth as it relates to lotteries focus on the regressive nature of the lottery. Since low-income players play a higher percentage of their income than do higher-income players, the lottery acts as a regressive taxing mechanism to generate revenue for the state. Refer to Chapter 11 of *Selling Hope: State Lotteries in America* (Clotfelter and Cook, 1989) for an overview of this issue.

7. A table of the states and games included in this study appears below. The number of observations for each game differs depending on which years the game was offered and when all variables had available values.

State	Game	State	Game
CO	Lotto	MN	Powerball
DC	Powerball	MO	Lotto
DE	Powerball	MO	Powerball
FL	Lotto	MT	Powerball
IA	Powerball	OH	Super Lotto
ID	Powerball	OR	Megabucks
IN	Powerball	OR	Powerball
KS	Powerball	SD	Dakota Cash
KY	Lotto Kentucky	SD	Powerball
KY	Powerball	WA	Lotto
MD	Lotto	WA	Quinto
ME	Powerball	WI	Powerball
MI	Lotto	WV	Powerball

8. Linear interpolation was also used to generate the inexplicably missing 1989 household numbers. Please contact the author for further information regarding these methods.
9. The demographics data was available from 1984-1994. This precluded recent games--Tri-West and The Big Game--from being part of the study. Also, none of the states involved in Tri-State Megabucks (Maine, Vermont, and New Hampshire) responded to requests for jackpot information, so that game could not be included.
10. Due to checking the regression for heteroskedasticity, the ordering is now by JACKPOT. Thus autocorrelation may still be present by the initial ordering. Later, in Model 4, an attempt to correct for possible remaining correlation due to regional differences was done by including regional dummies (NORTH CENTRAL, NORTHEAST, SOUTH, and WEST (excluded)). These regional dummies, however, did not improve the regression, and they detracted from its predictive powers. Thus they were not used in final results. Whether autocorrelation still exists or not, the author feels that statistical correction for two problems would not be appropriate; this study only employs weighted least squares.
11. The sample is divided into thirds and individual regressions are run. The small-JACKPOT third elicits a convex relationship, while the large-JACKPOT third elicits a concave relationship. This implies that the entire sample may be fit better by a cubic JACKPOT function.

12. As acknowledged in section III, a jackpot increase may actually *decrease* the demand by moving the potential winnings only further out on the same concave portion of a potential player's utility curve. If this idea explained the results, then it would require assuming many consumers' utility curves have an indivisibility point near the inflection point of the jackpot / revenue curve. Having considered the cubic relationship found, the author feels it is more plausible to believe that many consumers have a similar lack of interest due to repeated rollovers, as described in the paper, than to believe that consumers have similar indivisibility points.

REFERENCES

- Arrarte, Anne Moncreiff.** "Florida Lottery at a Turning Point, Seeking New Ideas." *The Miami Herald*, July 18, 1996.
- Aronson, J. Richard; Weintraub, Andrew; and Walsh, Cornelius.** "Revenue Potential of State and Local Public Lotteries." *Growth and Change*, April 1972, 3(2), pp. 3-8.
- Caudill, Steven B.; Gropper, Daniel M.; and Peng, Ter Chao.** "Predicting Net Revenues from State Supported Lotteries." *The Journal of Business Forecasting*, Spring 1995, 14(1), pp. 11-15.
- Clotfelter, Charles T. and Philip J. Cook.** *Selling Hope: State Lotteries in America*. Cambridge: Harvard University Press, 1989.
- _____. "On the Economics of State Lotteries." *Journal of Economic Perspectives*, Fall 1990, 4(4), pp. 105-119.
- _____. "The Peculiar Scale Economies of Lotto." *The American Economic Review*, June 1993, 83(3), pp. 634-643.
- Current Population Reports*, series P-20. Various issues.
- Current Population Reports*, series P-25. Various issues.
- Current Population Reports*, series P-60. Various issues.
- DeBoer, Larry.** "When Will State Lottery Sales Growth Slow?" *Growth and Change*, January 1986, 17(1), pp. 28-36.
- _____. "Lotto Sales Stagnation: Product Maturity or Small Jackpots?" *Growth and Change*, Winter 1990, 21(1), pp. 73-77.
- Friedman, Milton and Savage, L. J.** "The Utility Analysis of Choices Involving Risk." *The Journal of Political Economy*, August 1948, 56(4), pp. 279-304.

- Gulley, O. David and Scott Jr., Frank A.** "The Demand for Wagering on State-Operated Lotto Games." *National Tax Journal*, March 1993, 46(1), pp. 13-22.
- Illinois Economic and Fiscal Commission.** *The Illinois State Lottery: A Special Report*. Illinois: State of Illinois, 1987.
- Jackson, Raymond.** "Demand for Lottery Products in Massachusetts." *The Journal of Consumer Affairs*, Winter 1994, 28(2), pp. 313-325.
- Kahneman, Daniel and Tversky, Amos.** "Availability: A Heuristic for Judging Frequency and Probability," in *Judgment Under Uncertainty: Heuristics and Biases*, ed. Daniel Kahneman, Paul Slovic, and Amos Tversky. Cambridge: Cambridge University Press, 1982.
- _____. "Prospect Theory: An Analysis of Decision Under Risk." *Econometrica*, March 1979, 47(2), pp. 263-291.
- Knapp, Elaine S.** "Lotteries No Gamble." *State Government News*, March 1988, pp. 14-16.
- Kwang, Ng Yew.** "Why Do People Buy Lottery Tickets? Choices Involving Risk and the Indivisibility of Expenditure." *The Journal of Political Economy*, October 1965, 73(5), pp. 530-535.
- La Fleur's '96 World Lottery Almanac, Fourth Edition*, ed. Teresa La Fleur and Bruce La Fleur. Boyds, Maryland: TLF Publications, Inc., 1996.
- "Lotto Jackpots Dwindle." *Daily Herald*, 11 September 1996, Section 1, p. 1.
- Markowitz, Harry.** "The Utility of Wealth." *The Journal of Political Economy*, February 1952, 60(1)pp. 151-158.
- Meyers, Michelle J.** "Fiscal 1996 Sales Results." *Public Gaming International*, September 1996, pp. 8+.
- Mikesell, John L.** "The Effect of Maturity and Competition on State Lottery Markets." *Journal of Policy Analysis and Management*, Winter 1987, 6(2), pp. 251-253.
- Mikesell, John L. and Zorn, C. Kurt.** "State Lottery Sales: Separating the Influence of Markets and Game Structure." *Growth and Change*, Fall 1987, 18(4), pp. 10-19.
- Novak, Tim.** "Lottery's Big Game a Big Bust." *Chicago Sun-Times*, 26 March 1997, Section1, p.1.
- Ramanathan, Ramu.** *Introductory Econometrics with Applications*. Fort Worth: Harcourt Brace College Publishers, 1995.

- Scoggins, John F.** "The Lotto and Expected Net Revenue." *National Tax Journal*, March 1995, 48(1), pp. 61-70.
- Scott, Loren C.; Beard, Thomas R.; and Silvia Jr., William L.** *Should Louisiana Adopt a State Lottery?* Louisiana: Council of Economic Advisors, 1973.
- Sprowls, R. Clay.** "On the Terms of the New York State Lottery." *National Tax Journal*, March 1970, 23(1), pp. 74-82.
- Stover, Mark Edward.** "Revenue Potential of State Lotteries." *Public Finance Quarterly*, October 1987, 15(4), pp. 428-440.
- _____. "Contiguous State Lotteries: Substitutes or Complements?" *Journal of Policy Analysis and Management*, Winter 1990, 9(4), pp.565-568.
- Thiel, Stuart E.** "Policy, Participation and Revenue in Washington State Lotto." *National Tax Journal*, June 1991, 44(2), pp. 225-235.
- Vandeman, Scott.** Personal correspondence through e-mail, 17 February 1997.
- Vrooman, David H.** "An Economic Analysis of the New York State Lottery." *National Tax Journal*, December 1976, 29(4), pp. 482-489.
- Wu, Roland Y.** "A Cross-Section Study on the Demand for State Lottery Tickets." *American Economist*, Fall 1979, 23(2), pp. 6-12.