Economic Shocks, Trade and International Relations

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Abstract
In an interdependent world, trade has unavoidable game aspects. A model with two agents is used to determine the impact of trade and a military alliance between two major world players, North America and China, and an external non-actor, South Korea. The objective of this study is to investigate the impact of cooperative actions and outcomes by the two agents on a two-track policy for South Korea. We also study a variant to the game by considering a change in international relations. Welfare implications are also observed.

Keywords
Military alliance, sequential equilibrium, strategy, trade

Cover Page Footnote
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1. Introduction

In recent years, we can identify that South Korea’s long term economic interest is increasingly dependent on China with 2009 bilateral trade volumes increasing to 21.1% of total trade [source: IMF (DoTS)]. This rapid growth of bilateral trade has emerged by increased economic opportunity between the two countries, driven by the entry of China into the World Trade Organisation (WTO) in 2001. China’s entry into the WTO signified its acceptance of international rules of trade and enabled South Korea to invest\(^1\) in the Chinese economy [see Snyder and Byun (2010)]. A huge export market for South Korea developed. This strong international framework provided China and South Korea with a new mechanism for managing economic ambitions; the result being increased bilateral trade.

Progress in the interest of security however, is much more aligned with North America as we observe that China continues to provide a lifeline to North Korea. Defence spending in South Korea is crucial to provide a regional balance of power and thus, it is in the national interest of North America to do so [for example see Scalapino (1991)]. North America has both strategic and financial interests in South Korea. In 2003 North America was South Korea’s second-largest supplier of foreign direct investment [see Manyin (2004)]. For several years, North America and South Korea have been discussing a bilateral investment treaty (BIT) and there have been numerous calls to implement a free trade agreement (FTA), reinforcing the increased cooperation between the two powers [see Manyin (2004)]. North America’s economic partnership with China is obvious, with a 2010 trade balance of US$ -273,065.5 million\(^2\) [source: US Census Bureau]. We begin to see a pattern; international economic and political cooperation is needed. In this interdependent world, North America, China and South Korea are operating in a strategic environment and their actions depend on the policies of others; trading has unavoidable game aspects [see Ozyildibim (1996)].

This paper studies interactions between two nations, North America and China using an extensive form game where sequential rationality is assumed on the behaviour of each player [see Kreps and Wilson (1982)]. Despite its inescapability, few studies have attempted to formalize this line of argument. Krugman (1986) includes papers suggesting that there was a need for new thinking about trade policy, what it should target and the effects of such policies in an evolving international market. Brander (1986) considers strategic games played by national governments to welfare maximize and how

\(^1\) In 1992 South Korea’s direct investment in China was US$19 million and grew to $2.721 billion in 2002 – on average a 26.78 percent annual growth rate [see Zhan (2005)].

\(^2\) Works by Feenstra et al. (1998) and Fung and Lau (1998) question the size and determinants of these bilateral trade volumes, however, this is something we do not consider in this paper.
policy initiatives affect the state of play for other governments. Literature on 'Trade Wars' [for example see Conybeare (1987)] provided for interesting reading and lead me to consider the game aspects of international trade. Conybeare (1987) provided me with an initiative to study the politics of an interdependent world where the actions of one agent affect the strategy of another.

In Section 2 we formalize a game which is of complete but imperfect information, we call North America country 1 and China country 2. We call South Korea, who is an external non-actor, third country. I look for sequential equilibria and in Section 3 show the following result; for any \( q > 0.5 \), where \( q \) represents the state of country 1, which nature decides, country 1 plays \( U \) no matter what and country 2 plays \( A \) no matter what. This is the unique sequential equilibrium in game \( \Gamma \). This shows that the optimal strategy for the two countries is to trade with each other, and a third country. We also show that a military alliance between country 1 and a third country is welfare increasing for all countries.

The rest of this paper is set out as follows; in Section 4 we study an extension to game \( \Gamma \) by considering a change in international relations. A counter argument of that what is outlined above will be considered and will show the following result; for any value of \( q \), where \( q \) represents the state of country 1, which nature decides, country 1 plays \( D \) no matter what and country 2 plays \( C \) no matter what. This is the unique sequential equilibrium in game \( \Gamma_2 \). This shows that the optimal strategy for the two countries is to trade with each other as previously; but here country 1 not having a military alliance with a third country and country 2 not trading with a third country is welfare maximizing. The paper concludes in Section 5.

2. The Model

Consider the following extensive form game \( \Gamma \), shown in figure 1. In this game of complete but imperfect information there are two strategic players, country 1 (referred to in the Introduction as North America) and country 2 (referred to in the Introduction as China). There is an external non-actor, who we call third country (referred to in the Introduction as South Korea).

Nature sends a private signal selecting the state for country 1 which can be good or bad. Let \( \{q, 1-q\} \) denote a set of options about the state of the economy, where \( q \) stands for good state and \( 1-q \) stands for bad state. For the purpose of this paper we assume bad state is a recession. This information is not observed by country 2.
Let \( \{U, D\} \) be a set of strategies for country 1, independent of its private type of information, where \( U \) stands for the action of ‘trading with country 2 and providing a military alliance to a third country’ and \( D \) stands for the action of ‘trading with country 2 and providing no military alliance to a third country’. Let \( \{A, B, C, D\} \) be a set of strategies for country 2, where \( A \) stands for the action ‘acceptance of the offer by country 1 and trading with a third country’, \( B \) stands for the action ‘rejection of the offer by country 1 and trading with a third country’, \( C \) stands for the action ‘acceptance of the offer by country 1 and no trading with a third country’ and \( D \) stands for the action ‘rejection of the offer by country 1 and no trading with a third country’. We assume the level of trading between country 1 and a third country is external, remains constant and is positive throughout the game.

The equilibrium concept I look for is sequential equilibrium. Sequential equilibrium \([\text{see Kreps and Wilson (1982)}]\) is a solution concept based on the requirement of sequential rationality imposed on the behaviour of both countries. The strategic decisions of each country must be part of an optimal strategy. To examine optimality for country 2 we use Bayes' rule\(^3\) to attach probabilities to uncertain events at each information set of country 2.

We call these beliefs \( \delta, 1 - \delta, \gamma, 1 - \gamma \).

In game \( \Gamma \), \( \delta \) represents country 2’s belief that the state is good for country 1 upon observing \( U \). \( 1 - \delta \) represents country 2’s belief that the state is bad for country 1 upon observing \( U \). \( \gamma \) represents country 2’s belief that the state is good for country 1 upon observing \( D \). \( 1 - \gamma \) represents country 2’s belief that the state is bad for country 1 upon observing \( D \). The equilibrium concept used in this model is based on beliefs formed by country 2 about which information set he\(^4\) is at given the action of country 1, and the beliefs about what will happen in the future given his action.

\( \Gamma \) is played as follows; country 1 observes nature, which is either good or bad and at \( \alpha, 1 - \alpha, \beta \) and \( 1 - \beta \), which are defined below, country 1 moves. Let us suggest that country 1 plays \( U \); country 2 can play any strategy from set \( \{A, B, C, D\} \) which has to be optimal given his beliefs about the state, which is unknown to player 2. The same applies for when country 2 observes an offer of \( D \) by country 1. Hence we can say given \( \delta, 1 - \delta, \gamma \) or \( 1 - \gamma \) country 2 will decide what to do. At this stage in the game, country 1 now knows what country 2’s beliefs are and will choose an action from set \( \{\alpha, \beta\} \). Once country 1 does that, country 2 knows that country 1 knows what his beliefs are and therefore can compute back the actual value of \( \alpha \) and \( \beta \). This is how we find sequential equilibrium, which is dependent on payoffs.

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\(^3\) For theories of probability and its applications Bayes' rule shows the relationship between two conditional probabilities which are separate but dependable on each other [see Myerson (1991)].

\(^4\) The use of gender is arbitrary.
In game $\Gamma$, $\alpha$ is the actual probability with which country 1 plays $U$ when the state is good. $1 - \alpha$ is the actual probability with which country 1 plays $D$ when the state is bad. $\beta$ is the actual probability with which country 1 plays $U$ when the state is good. $1 - \beta$ is the actual probability with which country 1 plays $D$ when the state is bad.

The payoffs represent the motivations of the countries by ranking a set of outcomes. The payoffs for game $\Gamma$ are shown in figure 1. Let us normalise the payoff of action $\{U, A\}$ with state $q$ to $(8, 8)$; we suggest this because all countries are trading and the state is good for country 1. Let $\{U, D\}$ be a set action with state $q$ and let $(4, 4)$ be the payoff; we suggest this because although the state is good, no countries are trading. Following from this we can say action $\{U, B\}$ with state $q$ can be assigned payoff $(5, 7)$; reasoning for this reduction in utility for country 1 is because of the reduction in trade volumes. History shows bilateral trade between country 1 and country 2 is overwhelming, with a balance of US$ -273,065.5 million in 2010 [source: US Census Bureau], cutting this trade would be detrimental to country 1, shown by the reduced payoff. Utility of country 2 has also been reduced with this play. This can be explained by reduced export demand from country 1 but due to a surge in demand for Asian inter-regional trade [source: IMF], we suggest that utility has been reduced on a smaller scale. Action $\{U, C\}$ with state $q$ is assigned payoff $(9, 6)$; reasoning for this increase in utility for country 1 is because of the termination of trade between country 1 and a third country. This termination of trade reduces the amount country 2 can extort from a third country, so tomorrow, we can say that country 2 is less powerful. Country 1 is trading and has a military alliance with a third country, country 2 is doing neither; this is explained by the payoffs.
Let \( \{D, A\} \) be a set action with state \( q \) and let \( (7, 9) \) be the payoff; the increase in utility for country 2 can be explained by noting that country 1 is no longer providing a military alliance to a third country, so tomorrow we can say that country 1 is less powerful, shown by the decrease in utility. Let \( \{D, B\} \) be a set action with state \( q \) and let \( (4, 7) \) be the payoff; reasoning for this is that country 1 has no military alliance with a third country and is not engaging in trade with country 2 so utility is decreased. Utility here for country 2 is reduced for similar reasons to when action \( \{U, B\} \) in state \( q \) is played; trade is terminated. Let \( \{U, A\} \) be a set action with state \( q \) and let \( (8, 6) \) be the payoff; the increase in utility for country 2 can be explained by noting that county 1 is no longer providing a military alliance to a third country, so tomorrow we can say that country 1 is less powerful, shown by the decrease in utility. Let \( \{U, B\} \) be a set action with state \( q \) and let \( (4, 7) \) be the payoff; reasoning for this is that country 1 has no military alliance with a third country and is not engaging in trade with country 2 so utility is decreased. Utility here for country 2 is equal to the normalised value of when action \( \{U, A\} \) is played in state \( q \) because although there is no military alliance with this play, country 2 is not trading with a third country and so the effect on utility is equal to zero. Not trading with a third country reduces the utility to 6 for country 2 with this action. Let \( \{D, C\} \) be a set action with state \( q \) and let \( (8, 6) \) be the payoff. We can say that the utility of 8 here for country 2 is equal to the normalised value of when action \( \{U, A\} \) is played in state \( q \) because although there is no military alliance with this play, country 2 is not trading with a third country and so the effect on utility is equal to zero. Not trading with a third country reduces the utility to 6 for country 2 with this action. Let \( \{D, D\} \) be a set action with state \( q \) and let \( (4, 5) \) be the payoff; reasoning for this decrease in utility for country 1 is because at this action, although in a good state it is not trading with country 2 and it has no military alliance with a third country, so tomorrow, we can say that it is less powerful. Utility for country 2 is decreased because it is not trading with country 1, but utility is
higher than at set action \( \{U, D\} \) at state \( q \) because we suggest that it is able to extort more from a third country because there is no military alliance between country 1 and a third country.

Let \( \{U, A\} \) be a set action at state \( 1-q \) and let \( (5, 6) \) be the payoff; at this state country 1 is in a recession, so utility is decreased even though the trading position is the same as \( \{U, A\} \) at state \( q \). There is no assumption that country 2 is in a bad state, but we can say utility is decreased as a result of the bad state in country 1. Let \( \{U, B\} \) be a set action at state \( 1 - q \) and let \( (3, 7) \) be the payoff; here we observe country 1 is in a worse position because of the bad state combined with a termination of trade with its biggest trading partner. Utility of country 2 is reduced from the normalised value of 8 because of the result of a bad state and the reduced trade, but here we can say that the decrease in utility is of a smaller scale. We assume country 2 is able to finance its own economy, because its citizens are increasingly able to consume in large enough quantities to stimulate economic growth internally [see Abeyesinghe and Lu (2003)]. Let \( \{U, C\} \) be a set action at state \( 1 - q \) and let \( (5, 2) \) be the payoff; utility for country 1 has increased with this action because it is trading with country 2. Here, country 2 is not trading with a third country, suggesting it has reduced power in South-East Asia; this, combined with the military alliance between country 1 and a third country is reasoning for the larger decrease in utility for country 2. Let \( \{U, D\} \) be a set action at state \( 1 - q \) and let \( (4, 1) \) be the payoff; this represents the worst utility for country 2 because he is not trading with country 1 or a third country and country 1 has military alliance with a third country, so we can suggest that tomorrow, country 2 is less powerful. Utility for country 1 has decreased from action \( \{U, C\} \) at state \( 1 - q \) because there is no trade with country 2.

Let \( \{D, A\} \) be a set action at state \( 1 - q \) and let \( (4, 6) \) be the payoff; reasoning for this is because country 1 is in a bad state and it has no military alliance with a third country. The payoff here is less than \( \{U, A\} \) at state \( q \) simply because we assume country 1 is worse off tomorrow from not having a military alliance with a third country. The same applies for all the payoffs for country 1 in game \( \Gamma \). Let \( \{D', B\} \) be a set action at state \( 1 - q \) and let \( (1, 7) \) be the payoff; this action represents the lowest utility for country 1 because it is in a bad state, it has no military alliance with a third country and is not trading with country 2; furthermore country 2 is trading with a third country, which we suggest reduces the power further for country 1. We again suggest that country 2 has higher utility due to financing its internal economy and due to increased inter-regional trade. Let \( \{D, C\} \) be a set action at state \( 1 - q \) and let \( (4, 3) \) be the payoff; an increased utility for country 1 from set \( \{D, B\} \) at state \( 1 - q \) because of trading between country 1 and country 2; utility is still lower than the normalised payoff because of the state being bad and lack of military alliance with a third country. For country 2 utility has decreased on a greater scale because we suggest the reduced trade with a third country shows
reduced inter-regional trade. Let \( \{D', D\} \) be a set action at state \( 1 - q \) and let \( (2, 4) \) be the payoff; here utility has increased for country 1 action \( \{D, B\} \) at state \( 1 - q \) because although all the actions from country 1 are the same, country 2 does have the increased hold on a third country because it is not trading. Finally, we can see at this state of play, country 1 is in a bad state and none of the countries are trading which we may suggests shows a decline in external economic relations for all countries.

3. Analysis

We consider sequential equilibria. We will consider two cases separately: Can \( A \) be played no matter what is observed from set \( \{U, D\} \) constitute a sequential equilibrium? Can \( B \) be played no matter what is observed from set \( \{U, D\} \) constitute a sequential equilibrium? In the following theorem we show that for any \( q > 0.5 \) country 1 plays \( U \) no matter what and country 2 plays \( A \) no matter what; that is the unique sequential equilibrium in game \( \Gamma \).

The results of this model establish that country 1 and a third country trading and having a military alliance leads to efficient trading between country 1, country 2 and a third country. We suggest that this is a welfare maximizing strategy for South Korea because we can say that tomorrow due to the military alliance, China is less powerful and will therefore be able to extort less from South Korea. Corresponding literature on this centres around the work of Spero and Hart (2010) who conclude that the gap between international economics and politics is decreasing; when we consider trade, we need to consider the international political economy. We suggest that this equilibrium is welfare increasing for all countries. Chang-Hyung et al (2008) argue that China’s ambitions for a multilateral security framework would be welfare reducing for South Korea; from this we can conclude that North America and South Korea having a military alliance will suppress the power of China over South Korea, further supporting my results [for example see Synder and Byun (2010)].

Trade between North America and China has obvious welfare gains for both nations. North America has been able to invest in the expanding Chinese economy and take advantage of low-cost labour for exported goods [see Elwell and Labonte (2007)]. North American consumers have experienced increased purchasing power due to cheap Chinese imports. Finally, interest rates have remained relatively low due to China purchasing North American Treasury bonds, funding the federal deficit, allowing for growth [see Morrison and Labonte (2009)].

China joining the WTO has allowed bilateral trade between South Korea and China to increase by an average of 20 percent since 2001 [source:
KITA] due to increased political and economic security. Welfare gains of this increased cooperation are obvious. Direct investment of some 41,000 firms [source: KITA] from South Korea to China, including major corporate groups such as Samsung and LG have allowed South Korea to cultivate new industries and experience rapid growth.\(^5\) I now formalize these results.

**Theorem 1**  
For any \(q > 0.5\) country 1 plays \(U\) no matter what and country 2 plays \(A\) no matter what. Together these form the only sequential equilibrium in game \(\Gamma\).

Before we prove this theorem, we make the following observations natural for any sequential equilibrium.\(^6\) In this model, at each information set, country 2 must form a belief about the state of country 1, denoted by set \(\{q, 1 - q\}\). For game \(\Gamma\), a belief is in the form of a probability distribution over the nodes in the information set. Given country 2’s beliefs, his strategy must be sequentially rational. That is, no matter what country 2 observes from set \(\{U, D\}\), his action must be optimal given his beliefs about whether nature has selected good or bad for country 1 and his subsequent strategies. Given the action of country 2, the strategy of country 1 must be sequentially rational. That is, no matter what country 1 observes from set \(\{A, B, C, D\}\), his action must be optimal.

**Proof of Theorem 1**  
Upon observing \(U\), with beliefs \(\delta\) and \(1 - \delta\), country 2’s payoffs are as follows, given by utilities:

\[
\begin{align*}
U_2 (A | \delta, U) &= \delta.8 + (1 - \delta)6 = \delta.2 + 6 \\
U_2 (B | \delta, U) &= \delta.7 + (1 - \delta)7 = \delta.7 + 7 = 7 \\
U_2 (C | \delta, U) &= \delta.6 + (1 - \delta)2 = \delta.4 + 2 \\
U_2 (D | \delta, U) &= \delta.4 + (1 - \delta)1 = \delta.3 + 1
\end{align*}
\]

From this we can deduce that \(C\) and \(D\) will never be an optimal strategy for country 2 since any option from set \(\{A, B\}\) will generate higher levels of utility for any value of \(\delta\). This leaves two key findings regarding optimality for player 2 upon observing \(U\). \(A\) is an optimal strategy iff \(\delta.2 + 6 > 7\) and hence for when \(\delta > \frac{1}{2}\). Furthermore, \(B\) is an optimal strategy.

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\(^5\) 2010 average GDP growth was 6.1 percent [source: CIA (2011)].

strategy iff $7 > \delta \cdot 2 + 6$ and hence for when $\delta < \frac{1}{2}$. So when $\delta = \frac{1}{2}$, either $A$ or $B$ can be played. Note here that inequalities are sufficient because we are dealing with optimality.

Upon observing $D$, with beliefs $\gamma$ and $1-\gamma$, country 2’s payoffs are as follows, given by utilities:

$$U_2(A \mid \gamma, D) = \gamma \cdot 9 + (1-\gamma)6 = \gamma \cdot 3 + 6$$  \hfill (5)

$$U_2(B \mid \gamma, D) = \gamma \cdot 7 + (1-\gamma)7 = 7$$  \hfill (6)

$$U_2(C \mid \gamma, D) = \gamma \cdot 6 + (1-\gamma)3 = \gamma \cdot 3 + 3$$  \hfill (7)

$$U_2(D \mid \gamma, D) = \gamma \cdot 5 + (1-\gamma)4 = \gamma + 4$$  \hfill (8)

From this we can deduce that C and D will never be an optimal strategy for country 2 since any option from set $\{A, B\}$ will generate higher levels of utility for any value of $\gamma$. This leaves two key findings regarding optimality for player 2 upon observing $D$. $A$ is optimal iff $\gamma \cdot 3 + 6 > 7$ and hence for when $\gamma > \frac{1}{3}$. Furthermore, $B$ is an optimal strategy iff $7 > \gamma \cdot 3 + 6$ and hence for when $\gamma < \frac{1}{3}$. So when $\gamma = \frac{1}{3}$ either $A$ or $B$ can be played.

Consider country 2 playing $A$ with probability 1 when nature has selected that the state is good for country 1. Following our observations country 1 will always play $U$ and subsequently $\alpha = 1$. Using Bayes’ rule to attach a probability to this uncertain event at this information set of country 2, given by,

$$\delta = \frac{\alpha q}{\alpha q + \beta(1-q)}$$  \hfill (9)

we get $\delta = q$. To be consistent with player 2’s beliefs $\delta > \frac{1}{2}$ and this is true iff $q > \frac{1}{2}$. When nature has selected the state is bad for country 1, country 1 will always play $U$ then $\beta = 1$. Using Bayes’ rule to attach a probability to this uncertain event at this information set of country 2, given by,

$$1 - \delta = 1 - \left( \frac{\alpha q}{\alpha q + \beta(1-q)} \right)$$  \hfill (10)

we get $1 - \delta = 1 - q$ and so $\delta = q$. Therefore, for any $q > 0.5$ country 1 plays $U$ no matter what and player 2 plays $A$ no matter what; together these form a sequential equilibrium in game $\Gamma$.

We now show that the observations we made earlier suffice to eliminate our second case and allow us to rule out any other implausible equilibrium. Upon observing $U$, with beliefs $\delta$ and $1 - \delta$, country 2’s utilities are the same as previously. Here, however, we are considering that $B$ is the optimal strategy. We say $B$ is optimal iff $7 > \delta \cdot 2 + 6$ and hence for when $\delta < \frac{1}{2}$ and iff $7 > \gamma \cdot 3 + 6$ for when $\gamma < \frac{1}{3}$. So consider country 2 playing $B$.
with probability 1 when nature has selected the state is good for country 1. Following our observations country 1 will always play $U$ and subsequently $\alpha = 1$. We see again from Bayes’ rule that $\delta = q$ but for $B$ to be optimal if country 1 plays $U$, $\delta < \frac{1}{2}$ which is only true iff $q < \frac{1}{2}$ but when nature selects the state is good for country 1 $q > \frac{1}{2}$. This is not consistent with the beliefs of country 2 and hence there is no sequential equilibrium to support action $B$ upon observing $U$ at state $q$. The same is true for when nature selects the state is bad for country 1. $QED$

Theorem 1 shows that there is no other sequential equilibrium in game $\Gamma$. We now study a variant of this game and make suggestions about how a change in international relations affects equilibrium.

4. International Relations Revisited

We now study a variation of game $\Gamma$, we call it game $\Gamma_2$. This is shown in Figure 2. The methodology is the same as in my initial model, although the payoffs differ. The motivations behind these new payoffs are shown in Section 4.1. We study this game to consider a counter argument to the one expressed in $\Gamma$ and suggest that country 1 will get the highest utility when trading with country 2 and when not providing a military alliance with a third country, independent of state. Reasoning for this is because here we suggest that it is not efficient for North America to provide a military alliance as we assume that South Korea can be self-sufficient militarily against the North [see Niksch (2005)]. History supports this idea; in 2008 the number of North American troops in South Korea reduced to 28,500 [source: U.S. Department of State]. Plans by North America and South Korea to reduce the military alliance between the two nations came with an agreement to transfer wartime operational control to South Korea on April 17, 2012 [source: U.S. Department of State]. Further to this, South Korea’s defence budget rose from 2.8% of GDP in 2007 to 3.2% in 2008 [source: Council of Foreign Relations] suggesting that it would be economically inefficient for North America to continue to provide this one-sided military alliance to South Korea. We suggest that a military alliance damages the internal economic and political situation within North America due to increased hardship faced by North American soldiers in South Korea [for example see Bleiker (2003)].

We show that the optimal strategy for country 2 would be to trade with country 1 and to not trade with a third country because, although country 2 is a member of the WTO, there is no similar regional framework for facilitating political and economic issues between the two nations [see Snyder and Byun (2010)]. South Korea is not part of the China-ASEAN free trade agreement (ACFTA), a new diplomatic strategy of China which seeks to “capitalize on globalisation to accelerate China’s economic development and elevate
China’s power” [see Wang (2007) p.2]. South Korea not being a part of this leads us to suggest that China’s welfare is higher when dealing with countries inside ACFTA; reasoning for this is because of the geopolitical and geo-economic objectives\(^7\) of the FTA [see Wang (2007)].

To assess welfare implications for a third country, we look at the sequential equilibrium positions in game \(\Gamma\) and game \(\Gamma_2\) and use economic logic to assess the impact of such equilibria on South Korea. In this game, we suggest that South Korea in the long run will experience a welfare gain from not having a military alliance with North America, because it will be able to continue to build on an alliance with China, its regional partner. This will lead to China trading with South Korea, like in game \(\Gamma\) and South Korea will experience welfare gains from the net benefits this will create, both politically and economically. However, given the relations with North, the future of South Korea is uncertain.

We normalise the payoff of action \(\{U, B\}\) regardless of state to \((0, 0)\). This is because in this game, with a change in international relations we suggest that country 1 having a military alliance with a third country reduces utility because it is economically inefficient and causes economic and political unrest\(^8\); it is also not trading with country 2, its largest trading partner. For country 2, utility is also zero because we suggest that he gets a higher utility when he does not trade with a third country because of the absence of a regional trade agreement; with this action country 2 is not trading with country 1, further reducing utility.

\(^7\)Wang (2005) p.2 describes these as “cultivating good will among neighbours, maintaining regional stability and securing key markets and raw materials needed for China’s economic growth.”

\(^8\)More detailed explanations for the motivations behind these new payoffs, with corresponding literature have been outlined in Section 4.
Let \( \{U, A\} \) be a set action with state \( q \) and let \((1, 2)\) be the payoff; reasoning for this is because although both country 1 and country 2 are trading, country 1 has a military alliance with a third country and country 2 is trading with a third country, both of which are undesirable. The expected utility of country 1 is lower than that of country 2 because at state \( q \) we assume that country 1 having the military alliance with a third country is a waste of resources and could cause a further reduction in future utility.\(^9\) Let \( \{U, C\} \) be a set action at state \( q \) and let \((3, 4)\) be the payoff; this is the highest utility for both players when strategy \( U \) is played at state \( q \). Reasoning for this is because both countries are trading with each other and country 2 is not trading with a third country, given our assumptions about the economic climate in this game, this is the action which generates the highest utility for country 2. The utility of country 1 is lower than that of country 2 because it is still has a military alliance with a third country, which we assume to be undesirable. Let \( \{U, D\} \) be a set action at state \( q \) and let \((0, 1)\) be the payoff; utility has decreased here to the normalised value for country 1 because it is not trading with its largest trading partner and it is providing an inefficient military alliance to a third country. Reasoning for the utility of country 2 being higher than that of country 1 is the same as action \( \{U, A\} \) at state \( q \).

\(^9\) This assumption is made because we have established that South Korea is increasing its military spending as a percentage of GDP [source: Council of Foreign Relations] so North America supporting South Korea militarily is inefficient.
Let \( \{D, A\} \) be a set action at state \( q \) and let \((3, 2)\) be the payoff; reasoning for this is because both countries are trading and country 1 is not providing a military alliance to a third country. The payoff is lower for country 2 because he is trading with an external non-actor. Let \( \{D, B\} \) be a set action at state \( q \) and let \((1, 0)\) be the payoff; reasoning for this is the same as set action \( \{U, B\} \) at state \( q \) apart from the utility of country 1 is higher because he is not providing a military alliance to a third country. Let \( \{D, C\} \) be a set action at state \( q \) and let \((5, 5)\) be the payoff; this action generates the highest level of utility for both players in game \( \Gamma_2 \) because both countries are trading, country 1 does not have a military alliance with a third country nor is country 2 trading with a third country. Let \( \{D, D\} \) be a set action at state \( q \) and let \((2, 1)\) be the payoff; reasoning for this lower utility of country 2 is because at this action he is trading with a third country, which we suggest would be less efficient that country 2 trading with a member of ACFTA. Utility of country 1 has decreased at a lower rate because even though it is not trading with its largest trading partner, he gets utility from not having a military alliance with a third country.

Let \( \{U, A\} \) be a set action at state \( 1 - q \) and let \((1, 1)\) be the payoff; at this state country 1 is in a recession, so utility is decreased even though the trading position is the same as \( \{U, A\} \) at state \( q \). There is no assumption that country 2 is in a bad state, but we can say utility is decreased as a result of the bad state in country 1. With this action both countries are acting in ways which, with this economic climate they consider undesirable. Let \( \{U, B\} \) be a set action at state \( 1 - q \) and let \((0, 0)\) be the payoff; reasoning for this is the same as action \( \{U, B\} \) at state \( q \). Let \( \{U, C\} \) be a set action at state \( 1 - q \) and let \((2, 2)\) be the payoff; reasoning for this is because country 2 is acting optimally given our assumptions about international relations but because of country 1 being in a bad state, utility is lower for both countries than the same action at state \( q \). Let \( \{U, D\} \) be a set action at state \( 1 - q \) and let \((0, 1)\) be the payoff; reasoning for this is the same as action \( \{U, D\} \) at state \( q \). We can say here that the state of country 1 does not effect this set action.

Let \( \{D, A\} \) be a set action at state \( 1 - q \) and let \((2, 1)\) be the payoff; reasoning for this is because country 1, even though in a bad state, is acting optimally given our assumptions about international relations. Utility is lower for country 2 because it is not acting optimally given our assumptions; utility for both countries is lower than that of the same action at state \( q \) because of the effects of the recession affecting both countries. Let \( \{D, B\} \) be a set action at state \( 1 - q \) and let \((1, 0)\) be the payoff; reasoning for this is the same as set action \( \{D, B\} \) at state \( q \). We can say here that the state of country 1 does not effect this set action. Let \( \{D, C\} \) be a set action at state \( 1 - q \) and let \((3, 3)\) be the payoff; reasoning for this is the same as set action \( \{D, B\} \) at state \( q \) but here the state of the country 1 decreases the utilities for both countries because they are both trading with each other so there is a knock-on effect of
the bad state in country 1 on country 2. Finally, let \(\{D, D\}\) be a set action at state \(I - q\) and let \((I, I)\) be the payoff; reasoning for this is because although both countries are acting optimally given our assumptions about international relations, they are doing so when country 1 is in a bad state and therefore utility is reduced.

4.2 Analysis

Let us look at sequential equilibrium. We consider the case: Can \(C\) be played no matter what is observed from set \(\{U, D\}\) constitute a sequential equilibrium? In the following theorem we show that in game \(\Gamma_2\) for any value of \(q\) country 1 plays \(D\) no matter what and country 2 plays \(C\) no matter what; that is the unique sequential equilibrium.

The results of this extension show, due to a suggested change in international relations that country 1 trading with country 2 is optimal, but only when country 2 is not trading with a third country and country 1 does not have a military alliance with a third country. Reasoning for this is because of the lack of a regional framework between China and South Korea for managing the political economy aspects of trade [see Snyder and Byun (2010)]. We suggest that this may lead to inefficiencies due to trade wars; The Garlic War, The Kimchi War and The Melamine Scandal are some examples.\(^{10}\) Without the membership of South Korea in ACFTA, one can say that there isn’t willingness for both nations to develop stronger mechanisms for managing trade, and we may suggest that trade is inefficient and not an optimal strategy.

The optimal strategy for North America is to trade with China and to not provide a military alliance to South Korea. Reasoning for this trade with China is the same as in game \(\Gamma\), however, now we suggest that the added military alliance between North America and South Korea is inefficient. We say this because we assume that South Korea can be militarily self sufficient against the North [see Niksch (2005)].

**Theorem 2**  For any value of \(q\), country 1 plays \(D\) no matter what and country 2 plays \(C\) no matter what. Together these form the only sequential equilibrium in game \(\Gamma_2\).

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\(^{10}\) Snyder and Byun (2010) evaluate these in some detail.
Proof. To establish this theorem we make the same observations as in game \( \Gamma \). Upon observing \( U \), with beliefs \( \delta \) and \( 1 - \delta \), country 2’s payoffs are as follows, given by utilities:

\[
\begin{align*}
U_2 (A \mid \delta, U) &= \delta \cdot 2 + (1 - \delta) \cdot 1 = \delta + 1 \\
U_2 (B \mid \delta, U) &= \delta \cdot 0 + (1 - \delta) \cdot 0 = 0 \\
U_2 (C \mid \delta, U) &= \delta \cdot 4 + (1 - \delta) \cdot 2 = 2 \cdot \delta + 2 \\
U_2 (D \mid \delta, U) &= \delta \cdot 1 + (1 - \delta) \cdot 1 = 1
\end{align*}
\]

(11)  (12)  (13)  (14)

For \( C \) to be optimal \( 2 \cdot \delta + 2 > \delta + 1; 2 \cdot \delta + 2 > 0 \) and \( 2 \cdot \delta + 2 > 1 \); this will always hold. From this we can deduce that any from set \( \{A, B, D\} \) will never be an optimal strategy for country 2 since strategy \( C \) will always be optimal given any value of \( \delta \).

Upon observing \( D \), with beliefs \( \gamma \) and \( 1 - \gamma \), country 2’s payoffs are as follows, given by utilities:

\[
\begin{align*}
U_2 (A \mid \gamma, D) &= \gamma \cdot 2 + (1 - \gamma) \cdot 1 = \gamma + 1 \\
U_2 (B \mid \gamma, D) &= \gamma \cdot 0 + (1 - \gamma) \cdot 0 = 0 \\
U_2 (C \mid \gamma, D) &= \gamma \cdot 5 + (1 - \gamma) \cdot 3 = 2 \cdot \gamma + 3 \\
U_2 (D \mid \gamma, D) &= \gamma \cdot 1 + (1 - \gamma) \cdot 1 = 1
\end{align*}
\]

(15)  (16)  (17)  (18)

For \( C \) to be optimal \( 2 \cdot \gamma + 3 > \gamma + 1; 2 \cdot \gamma + 3 > 0 \) and \( 2 \cdot \gamma + 3 > 1 \); this will always hold. From this we can deduce that any from set \( \{A, B, D\} \) will never be an optimal strategy for country 2 since strategy \( C \) will always be optimal given any value of \( \gamma \).

Consider country 2 playing \( C \) with probability 1 when nature has selected that the state is good for country 1. Following our observations country 1 will always play \( D \) and subsequently \( \alpha = 0 \). Using Bayes’ rule,

\[
\gamma = \frac{(1 - \alpha)q}{(1 - \alpha)q + (1 - \beta)(1 - q)}
\]

(19)

we get \( \gamma = q \). This is consistent with country 2’s beliefs for any value of \( \gamma \). When nature has selected the state is bad for country 1 he will always play \( D \) and then \( \beta = 0 \). Using Bayes’ rule,

\[
1 - \gamma = \frac{(1 - \alpha)q}{(1 - \alpha)q + (1 - \beta)(1 - q)}
\]

(20)

we get \( 1 - \gamma = 1 - q \) and so \( \gamma = q \). This is consistent with country 2’s beliefs for any value of \( \gamma \). Therefore, we can say for any value of \( q \) country 1
plays D no matter what and country 2 plays C no matter what together form the only sequential equilibrium in game $\Gamma_2$. [QED]

5. Concluding Remarks

In this paper we have considered a strategic analysis of trade and focused on sequential equilibrium [see Kreps and Wilson (1982)] by constructing a game of complete but imperfect information to analyse trade patterns between two countries and a third country, who is an external non-actor. I also consider what impact a military alliance between country 1 and this third country has on equilibrium. I have shown that for country 2, any option from set $\{C, D\}$ will never be optimal and so the strategy disappears in game $\Gamma$. It follows from theorem 1 that for any $q > 0.5$ country 1 plays $U$ no matter what and player 2 plays $A$ no matter what together form the only sequential equilibrium in game $\Gamma$.

From what I have established in this paper, I can conclude that country 1 and a third country having a military alliance leads to efficient trade relations between country 1, country 2 and a third country. This is proved formally in game $\Gamma$. Reasoning for this is because we can say that country 1 having a military alliance with a third country reduces the amount that country 2 can extort from this third country and tomorrow, it is less powerful. Although the third country is external, we may suggest that it will experience the largest gains in welfare by having a military alliance with country 1 and trading with both countries; effectively having balanced relations between the two countries.

I consider an extension to this model by studying a change in international relations. It follows from theorem 2 that for any value of $q$ country 1 plays $D$ no matter what and country 2 plays $C$ no matter together form the only sequential equilibrium in game $\Gamma_2$.

An extension of this model in the direction of analysing the differences in sequential equilibrium for when the military alliance between country 1 and an external third country is secretive would be desirable. However, the aim of this study is to introduce sequential equilibrium when a military alliance is observed rather than country 2 having to form a belief about this. Nevertheless, if the payoffs are known, the adjustments to this model would be straightforward.
References


