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A Household Model of Careers and Education Investment

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Abstract

This paper develops a two-stage non-cooperative household game, in which parents make career decisions and an investment into their child's human capital. The model is solved for Nash equilibrium outcomes and extended for a cooperative solution. In non-cooperative pure strategies, both parents choosing to work is a Nash equilibrium, though there are alternative outcomes when the conditions underlying the career decision are varied. The investment behaviour of agents is analysed. We find that choices are critically affected by the magnitude of the cost (and reflected quality) of a high education investment relative to a low investment, and the intrinsic value that a parent places on the child's future success. The cooperative game demonstrates parents maximise their investment in order to provide their child with a high quality education.

Keywords

Game theory, bargaining, household, education

1. Introduction

Household decision-making has been explored for decades, investigating an array of recurrent themes; among them, gender, education, marriage, fertility, divorce, labour, and domestic violence. We are interested in the conditions under which parents in a household will choose to work and said household's optimal investment in the education of children. In this section, the popular literature is outlined. We then establish the aims and structure of this paper.

Modelling rational families in limitless environments is considerably abstract. Nevertheless, understanding the behaviour of families is important for developing policies and appropriate institutions. Simple modelling formulates decision-making mechanisms and ideals for society to strive towards in the long run. The literature as a whole appears to ratify this view, and illustrates many techniques used to model household decisions, from as early as the 1960s. Due to the theoretical nature of the subject, empirical evidence is found but not widely explored. Xu (2007) identifies three prevalent approaches of the literature.

Firstly, Nash bargaining models, eminently developed by Manser and Brown (1980) and McElroy and Horney (1981). In these models, outcomes are always efficient, and there exists a threat point, beyond which the marriage between two players will dissolve. Whilst these models have important empirical implications, they cannot be easily tested with observable data. Choosing an appropriate threat point is particularly difficult in an entirely theoretical model, as is identifying the 'threat' itself, which is often unrealistic and external to the marriage scenario.

Secondly, there are collective models (or Pareto-efficient models), which depict the family as a single decision-making unit, rather than a group of independent agents. Chiappori (1988a, 1997) characterises these, taking a general approach typically with minimal assumptions, to be widely-applicable. Consequently, no specific games or decision-making processes within the household are specified. Similar to Nash bargaining, the outcomes are efficient.

Thirdly, non-cooperative bargaining models have been developed as a result of academic dissatisfaction with the limitations of Nash-cooperative models. It is important to note that those developed so far are not entirely non-cooperative; development of this concept still needs to make significant progression in order to withstand empirical scrutiny. Parents within the household do not enter binding contracts with each other, but instead behave as sovereign entities within the household economy. It is important to note that

those developed so far are not entirely non-cooperative; development of this concept still needs significant progression to withstand empirical scrutiny.

Becker, a clear pioneer in household modelling, typically treats the family as a single decision-maker (Becker, 1981), with either homogeneous preferences across all members or one all-powerful leader who decides for all. Members of a household pool their income, and maximise a household welfare collectively. This is disputed by Browning and Chiappori (1998) amongst others, who reject income pooling for two-person households. In the context of developing countries, where families are often headed by a single male earner, perhaps a unitary analogy would be loosely appropriate. However, for developed countries, this idea on average is less applicable. After all, women comprise an increasing majority of the labour force in OECD countries[1].

In conjunction with declining fertility and growing independence of women, the unitary approach is increasingly rejected across the literature, notably by Thomas (1990). Thomas finds the impact of unearned income differs between spouses; for example, income spent by mothers on health and nutrition improves the probability of a child surviving by twenty-fold that of expenditure by fathers. Thomas' objection is justified because there is incentive for spouses to deviate from operating collectively, and instead allocate their individual resources to goods and outcomes they prefer.

Intra-household bargaining begins in the literature in the 1970s and was significantly developed by Chiappori (1988b), who vindicated abandoning Nash bargaining altogether, and agents making Pareto-efficient decisions collectively as a unit. Chen and Woolley (2001) use Cournot-Nash equilibrium as the threat point in their bargaining game, and find this yields a Pareto-inferior result. Due to the extremities in fairness of Pareto efficiency[2], it is debatable whether this is a strength or a weakness.

Alongside variations in structural approach are the wide variety of issues investigated in the literature. Surprisingly popular is the effect of commitment on decision making. Rasul (2008) introduces this notion, comparing fertility bargaining outcomes in Malaysia with and without commitment. The literature examining fertility considers the decision to have children, and how many to have. Rasul (2008) also investigates how fertility preferences translate into fertility outcomes in a bargaining model, with his utilisation of Malaysian household survey data.

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[1] Source: OECD Employment and Labour Market Statistics.

[2] A resource distribution is Pareto-efficient if no individual can be made better off without making another worse off. Thus it is possible for a distribution to be such that one individual is allocated everything, and all others nothing. Pareto efficiency therefore does not mean equality or fairness.

Developing countries are a popular focus of recent literature. There are many analyses of nutrition; in particular, expenditure on nutrition, and the nutrition of children. Park (2007) explores children's nutrition and education in Indonesia, finding mothers have greater bargaining power over the nutrition of children, but that spouses are more equal when deciding upon education investment. Basu, Das and Dutta (2007) scrutinise child labour, and find child labour in developing countries increases with land wealth until a turning point when the relationship reverses. Whilst this paper does not address development, the comparisons are useful for interpreting results, and perhaps of interest for future study.

There is a small concentration of higher education analyses, with concentration on the rates of return to higher education. Welch (2007) compares public and private higher education in Indonesia, discussing a continued trend towards private higher education institutions. This paper's concern is only with education in the early life of a child, and would maintain that early education is more critical than higher education. We would argue that early education is an important determinant for the success of a child. Higher education is a later step in pursuit of this success, thus the two could be better investigated together.

The choice of private versus public school is seldom raised in the literature. Rather than being an irrelevant subject for discussion, there are more significant factors that are alternatively investigated, such as income, the availability of education choices, and the quality of these choices. For instance, Dustmann (2004) provides a detailed analysis of school track choice in Germany, and how it is primarily influenced by parental background and a child's gender. In Germany, there are few private schools, and those that exist are mandated to charge very low fees. As will be shown in this analysis, and as one would expect, the relative cost of private to public education is a critical determinant of parents' school choice. Oppositely at a macroeconomic level, Glomm and Ravikumar (1992) consider public and private education in relation to income, and their positive effect on a country's growth and equality. Similarly, Pomponio and Lancy (1986) contrast public education investment of developing country governments to investment in private education by parents.

Though the impact of a child's gender on their parents' expenditure is not explored in this paper, it is a common theme across the literature. This has important implications for our results, likely to alter the relative importance of a child's welfare in parents' utilities, dependent upon gender. Echevarria and Merlo (1999) find gender differences in education are smaller when consumption is included in the utility function, rather than a pure investment model, and Bonke and Esping-Andersen (2007) determine male children as an important requisite for a father's time.

The impact of divorce on a child's welfare is not considered here, but has interesting connotations for inter-generational income redistribution. Del Boca and Flinn (1994) consider divorced parents' expenditures on children; their work is extended by Nunley and Seals (2009), who examine how child custody affects investment in children's private school education.

In order to give real context to this paper, it is useful to consider the empirical environment for its application. The issues at hand are parents, their decisions to work, and their investment in children.

The perception of men and women in households varies between the developed and developing world, and across cultures and societies. It is therefore impossible, and arguably inappropriate, to wholly define the conventions of each gender, particularly since this paper has not been modelled on a particular social group. We can however draw upon prevalent observations of adopted gender roles across the demographic literature, in order to make practical conclusions.

Oaxaca (1973) argues women's employment structures differ to those of men due to labour market discrimination. Women plan to work less because discrimination renders them with lower returns, in the form of lower wages. We have not accounted for labour market discrimination, but it is an important issue for our results, as this would likely distort women's incentive to work, and thus the career choice of a female spouse.

Fortin (2005) stereotypes women as homemakers and men as breadwinners in OECD countries. Although this paper does not explicitly adopt this view, differing roles of agents in a household have been accounted for through heterogeneity.

This paper looks to explore the choices made by parents in the early life of a child; their decisions to work and their individual investments in the child's education, and how this is affected by their resources and preferences over the qualities of a child. We develop a multi-stage non-cooperative game with a cooperative extension in order to determine the optimal choices for heterogeneous spouses. We show the following. In a non-cooperative environment, where agents operate independently, it is always optimal for both parents to invest highly in the child's human capital (education). There are two attainable career outcomes in pure strategy equilibrium – either both parents work, or the parent that prioritises education works, whilst the other raises children at home. Our cooperative analysis shows that parents will jointly pay for the highest quality education available to maximise the welfare of a child.

The rest of the paper is structured as follows. In section 2, we formulate the model. In section 3, the model is manipulated to demonstrate the existence of

Nash equilibrium investment outcomes. In section 4, the analysis and results are interpreted and applied to find optimal career paths. In section 5, we briefly explore an alternative type of household through a cooperative extension. Lastly in section 6, we conclude.

2. The model

There are two players, representative of spouses who have, or who are contemplating having, a child. The spouses must choose career paths and this decision will impact their level of investment in the human capital of their child. We are assuming agents do not communicate, and instead behave according to their best responses.

Consider the stage 1 normal form game, Γ_1.

		Player 2	
		W	*H*
Player 1	*W*	NE_{WW}	NE_{WH}
	H	NE_{HW}	NE_{HH}

Each player independently makes a career decision; whether to work (*W*), or to not work and stay at home (*H*).

The stage 2 sub-game Γ_C subsequently depends upon the household's career outcome of Γ_1. There are four possible combinations and consequently, there will be four Γ_C sub-games of the combined work decision:

$$\mathbf{C} = \{WW\ WH, HW, HH\}$$

Having made this career choice, player *i* now independently chooses $x_i \in \{h, l\}$ – whether to make a high $(x_i = h)$ or low $(x_i = l)$ non-negative investment in their child's education.

C		Player 2	
		H	*l*
	h	$U_i(h,h \mid C)$	$U_i(h,l \mid C)$
Player 1	*l*	$U_i(l,h \mid C)$	$U_i(l,l \mid C)$

Human capital investment is regarded as a public good, either provided by parents or not, and non-rivalrous and non-excludable. The utility parents derive from investing in education is not affected by the presence of the other investing parent.

Players make decisions that maximise their individual payoffs subject to their partner's choice. They derive utility from their individual disposable income $(y_i - x_i)$, the household's total investment in education $(x = x_1 + x_2)$, and some intrinsic happiness from the emotional 'quality' of their child, e.

Total household investment in education x can be interpreted as the utility a parent receives from their child's 'output potential'; the success and income they will be able to achieve as an adult as a direct manifestation of their education. Therefore, parents can contribute to their child's wellbeing (and subsequently, their individual utility) in two ways; by providing them with an education and by developing their emotional quality and stability. We assume parents can make either of these investments without risk; the child's fruition and therefore the parents' happiness are guaranteed.

Players are selfish, and do not derive any utility from their partner's well-being. This may seem an unrealistic assumption in a marriage situation, where we would typically expect spouses to display altruism toward each other[3]. However, it is acceptable for this paper's purposes; aside from their own, the child's welfare is of primary focus of each player here, rather than that of their spouse, who should not be elemental in a human capital investment decision. Utility for both players is increasing in their individual investment. For simplicity, though common in general household literature, private consumption has not been included here in the payoff function.[4]

The players' general payoffs are then as follows:

$$\begin{aligned} U_1(x_1, x_2 \mid C) &= y_1^C - x_1 + x^2 + e \\ &= y_1^C - x_1 + (x_1 + x_2)^2 + \frac{y_1^C + y_2^C + t_2^C}{2} \\ U_2(x_1, x_2 \mid C) &= y_2^C - x_2 + \theta x + e^2 \\ &= y_2^C - x_2 + \theta(x_1 + x_2) + \left(\frac{y_1^C + y_2^C + t_2^C}{2}\right)^2 \end{aligned}$$

For realism purposes, we assume players have different utility functions. Collective unitary models assume homogeneity of preferences across agents, thus heterogeneous spouses seem a natural feature for a non-cooperative model and are common in the literature. Further reasoning for heterogeneous utility functions will be given in section 4.

[3] Ermisch (2003) defines altruism in the economic context to describe when an individual's welfare is dependent on the welfare of anfrom the utility of the child.

[4] Although irrelevant to this investigation, for argument's sake personal consumption could be interpreted as implicit in y_i, if y_i is defined as income net of private expenditure.

It is important to note the payoffs are affected by the career outcome of Γ_1, **C**. This affects whether the player earns an income y_i and the time spent at home by player 2 should they choose to not work t_2. For instance, if player *i* chooses to work, $y_i > 0$; if player *i* chooses not to work, $y_i = 0$. If player 2 chooses to work, $t_2 = 0$; if player 2 chooses not to work, $t_2 > 0$. 'Not working' could even be likened to the maintenance of an undemanding, part-time job. This would allow the parent in question to spend sufficient time at home to be emotionally beneficial to the child, whilst earning a negligible income that would not affect utility.

Weight θ is used in player 2's utility function to represent heterogeneous preferences. θ is the relative value that player 2 places upon the child's education and the subsequent (and guaranteed) success of the child attributed to their education. A larger value of θ indicates player 2 believes education is an essential driver of success; a smaller value of θ implies that player 2 thinks education is a less-significant contributor to success.

The utility component *e* represents the child's emotional stability and development. It is a function of the household's income $y_1 + y_2$, and the time spent at home by parent 2 raising the child should they choose not to work t_2. *e* is also a public good. Both parents derive utility from the emotional quality of the child; this utility is not affected by the presence of another parent. It is reasonable to infer that time spent at home with the child by one parent would contribute to their emotional wellbeing. For now, we assume the selection of parent 2 is arbitrary. Furthermore, we assume that players are always able to make an investment, regardless of their individual income. Even if a player chooses not to work, they are able to find money from some undefined source (perhaps savings, parents, inheritance, other assets). Therefore, individual investment in the child's human capital will always be non-negative, $x_i \geq 0$.

In this paper, the child is not considered explicitly as in other models. Their existence and welfare is integrated into the parents' utility. Young children have no decision-making power or skills to enable them to earn an income, thus their exclusion as players is without loss of generality.

Given the defined utility functions, the four **C** sub-games are depicted below.

{W,W}

Player 2

		h	l
Player 1	h	$y_1 - h + 4h^2 + \frac{y_1 + y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - h + (h + l)^2 + \frac{y_1 + y_2}{2}$, $y_2 - l + \theta(h + l) + \left(\frac{y_1 + y_2}{2}\right)^2$
	l	$y_1 - l + (h + l)^2 + \frac{y_1 + y_2}{2}$, $y_2 - h + \theta(h + l) + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - l + 4l^2 + \frac{y_1 + y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2$

{W,H}

Player 2

		h	l
Player 1	h	$y_1 - h + 4h^2 + \frac{y_1 + t_2}{2}$, $-h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2$	$y_1 - h + (h + l)^2 + \frac{y_1 + t_2}{2}$, $-l + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2$
	l	$y_1 - l + (h + l)^2 + \frac{y_1 + t_2}{2}$, $-h + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2$	$y_1 - l + 4l^2 + \frac{y_1 + t_2}{2}$, $-l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2$

{H,W}

Player 2

		h	l
Player 1	h	$-h + 4h^2 + \frac{y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2$	$-h + (h + l)^2 + \frac{y_2}{2}$, $y_1 - l + \theta(h + l) + \left(\frac{y_2}{2}\right)^2$
	l	$-l + (h + l)^2 + \frac{y_2}{2}$, $y_2 - h + \theta(h + l) + \left(\frac{y_2}{2}\right)^2$	$-l + 4l^2 + \frac{y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2$

{H,H}

		Player 2	
		h	l
Player 1	h	$-h+4h^2+\frac{t_2}{2}$, $-h+2\theta h+\left(\frac{t_2}{2}\right)^2$	$-h+(h+l)^2+\frac{t_2}{2}$, $-l+\theta(h+l)+\left(\frac{t_2}{2}\right)^2$
	l	$-l+(h+l)^2+\frac{t_2}{2}$, $-h+\theta(h+l)+\left(\frac{t_2}{2}\right)^2$	$-l+4l^2+\frac{t_2}{2}$, $-l+2\theta l+\left(\frac{t_2}{2}\right)^2$

The model is solved by backward induction, first finding a unique Nash equilibrium (NE) investment outcome in each Γ_C sub-game. Γ_1 is then solved for the NE career path of the players, taking the previously determined equilibrium investment choices as given.

Our first objective is to find a NE education investment outcome in each sub-game; the sub-game perfect equilibrium of Γ_C.

3. Equilibrium investment in education

The aim is to find NE of the players' career and investment decisions. In order to find a sub-game perfect equilibrium, we firstly consider Γ_C in its general form, where letters A-H represent the payoffs of the players.

Γ_C

		Player 2	
		h	l
Player 1	h	A , B	C , D
	l	E , F	G , H

For clarity, throughout this paper we assume $l = 0$, and $h > l$. Under these conditions, we find the NE total level of investment, given career choice **C**, is critically affected by the relative magnitude of h to l, and the value of weight θ in player 2's utility function. The investment outcome of Γ_C is determined by h and θ. We find that all outcomes can be sustained as pure strategy equilibria under different conditions. The conditions ensure that players will not deviate from each outcome.

Theorem 1. *Assume* $\theta > 1$.

(i) *When* $0 < h < \frac{1}{3}$, *(l, h) is the unique NE of all sub-games.*
(ii) *When* $h > \frac{1}{3}$, *(h, h) is the unique NE of all sub-games.*
(iii) *When* $h = \frac{1}{3}$, *there is no NE in pure strategies.*

Proof.

(i) For *(l, h)* to be a NE in Γ_C, it must hold that E > A and F > H.

E > A:

$$y_1 - l + (h+l)^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - h + 4h^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h < \tfrac{1}{3}$$

F > H:

$$y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - l + 2\theta l + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad \theta > 1$$

These conditions ensure that *(h, h)* and *(l, l)* are not NE.

For *(l, h)* to be a unique NE in Γ_C, *(h, l)* also cannot be a NE.
It must hold that G > C or B > D.

G > C:

$$y_1 - l + 4l^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - h + (h+l)^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h < 1$$

B > D:

$$y_2 - h + 2\theta h + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad \theta > 1$$

Given that this has been deduced from the general form of **C**, we can observe these conditions are true for all **C**.

(ii) For *(h, h)* to be a NE in Γ_C, it must hold that A > E and B > D.

A > E :

$$y_1 - h + 4h^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - l + (h+l)^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h > \tfrac{1}{3}$$

B > D:

$$y_2 - h + 2\theta h + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad \theta > 1$$

These conditions ensure that *(l, h)* and *(h, l)* are not NE.

For *(h, h)* to be a unique NE, *(l, l)* also cannot be a NE.
It must hold that C > G or F > H.

C > G:

$$y_1 - h + (h+l)^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - l + 4l^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h > 1$$

F > H:

$$y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - l + 2\theta l + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad \theta > 1$$

The condition 'C > G' requires that both $h > \frac{1}{3}$ and $h > 1$. Only 'C > G' or 'F > H' is required to uphold *(h, h)* as a unique NE in pure strategies. For ease, we impose the latter as sufficient; 'C > G' is only an additional requirement, and can be ignored.

As before, this is true for all **C**.

(iii) When $h = \frac{1}{3}$, and $\theta > 1$, a true NE cannot be reached unless player 1 is endowed with preferences such that in the event of indifference, he or she prefers making one investment over the other. No such preference relation has been granted. We can also infer that $h = \frac{1}{3}$ is a (near) zero-probability event, as probability is defined on a continuous scale. It is included as a proof only to be exhaustive. ■

Theorem 2. *Assume* $\theta < 1$.
(i) When $0 < h < 1$, *(l, l) is a unique NE of all sub-games.*
(ii) When $h > 1$, *(h, l) is a unique NE of all sub-games.*

Proof.

(i) For *(l, l)* to be a NE in Γ_C, it must hold that G > C and H > F.

G > C:

$$y_1 - l + 4l^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - h + (h+l)^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad 1 > h$$

H > F:

$$y_2 - l + 2\theta l + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - h + \theta(h + l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad 1 > \theta$$

These conditions ensure that *(h, l)* and *(l, h)* are not NE.

For *(l, l)* to be a unique NE, *(h, h)* also cannot be a NE.
It must hold that E > A or D > B.

E > A:

$$y_1 - l + (h + l)^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - h + 4h^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad \tfrac{1}{3} > h$$

D > B:

$$y_2 - l + \theta(h + l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - h + 2\theta h + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad 1 > \theta$$

The requirements for *(l, l)* to be a unique NE specify that $h < 1$ and $h < \frac{1}{3}$. Only 'E >A' or 'D > B' is required for *(l, l)* to be a unique NE so, for simplicity, we assume the latter. 'E > A' is unimportant.

This is true for all **C** outcomes.

(ii) For *(h, l)* to be a NE in Γ_C, it must hold that C > G and D > B.

C > G:

$$y_1 - h + (h + l)^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - l + 4l^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h > 1$$

D > B:

$$y_2 - l + \theta(h + l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - h + 2\theta h + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$
$$\Leftrightarrow \quad 1 > \theta$$

These conditions ensure that *(h, h)* and *(l, l)* are not NE.

For *(h, l)* to be a unique NE, *(l, h)* also cannot be a NE.
It must hold that A > E or H > F.

A > E:

$$y_1 - h + 4h^2 + \frac{y_1 + y_2 + t_2}{2} \quad > \quad y_1 - l + (h + l)^2 + \frac{y_1 + y_2 + t_2}{2}$$
$$\Leftrightarrow \quad h > \tfrac{1}{3}$$

H > F:

$$y_2 - l + 2\theta l + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2 \quad > \quad y_2 - h + \theta(h + l) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$

$$\Leftrightarrow \quad 1 > \theta$$

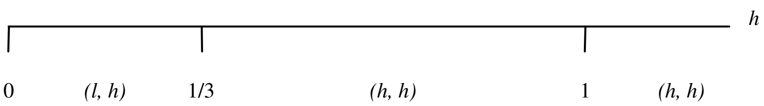
This holds for all **C**. ■

Theorem 3. *When* $\theta = 1$*, there is no NE in pure strategies.*

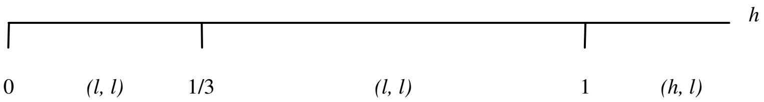
Proof. When $\theta = 1$, player 2 will always be indifferent between making investments *h* and *l*. This result can only be violated if assumptions are made about player 2's preferences over *h* and *l*. For example "For player 2, *h* is strictly preferred to *l*"; therefore when indifferent, player 2 will choose *h*. As no such statements have been made, NE cannot be reached under this condition in pure strategies; it is a (near) zero-probability outcome. Its inclusion is merely for thoroughness. ■

These findings can be summarised in a scale diagram for investment level *h*.

θ > 1:



θ < 1:



All NE investment outcomes are possible under different conditions. The implications of these conditions will be realised in the next section.

4. Interpretation and results

The analysis is now explored to offer some palpable and applicable explanation. We decipher the investment choices of each player under all conditions, and use these to determine under which circumstances it is optimal for parents to work or to stay at home.

Remark: So far we have maintained androgyny but it could be argued appropriate to identify player 1 as male and player 2 as female by our heterogeneous utility functions. Player 2 places greater worth on the emotional quality of the child than player 1. Becker (1981) claims that as a consequence of a greater biological investment by a mother into a child's upbringing through carrying and nursing, it is natural for her to want to see her investment bear fruit. Further, married women traditionally specialise in household and child-rearing activities, as they hold comparative advantage in these fields. This could be applied to player 2, who has more incentive (utility) to maximise the emotional stability and development of the child due to this sunk cost. A lower θ could also be interpreted as a reflection of societal customs. These are the social conventions that arguably make it easier for women to substitute careers for child-bearing activities. Some examples are maternity leave, variable work schedules (Flexitime), sympathetic attitudes, and the provision of childcare. Nevertheless, this perception is entirely at the discretion of the reader.

4.1 Investment choices

It was previously found that each investment combination could be reached as pure strategy NE outcomes in the investment sub-games under certain conditions. Two types of human capital investment, 'high' and 'low', have been distinguished. This can be likened to parents making a high investment and sending a child to a fee-charging private school at significant cost or opting for a state-provided 'free' education.

It is sensible to assume that the high investment cost of private education is a reflection of its quality. The higher h, the better quality of education the child will receive. Consequently, parents will derive greater utility from sending a child to a private school, everything else constant, as this will contribute more to the child's future success.

Notice that the structure of our game implicitly highlights two categories of private school. When one parent invests highly in the child's education and one parent invests low, $x = h + l$. Assuming $l = 0$, $x = h$. The household can afford in their investment allocation to education to pay tuition fees and send the child to an independent school. Such a school will provide the child with a better quality of education than a state school, perhaps due to smaller class sizes, highly qualified teaching staff and better facilities. When both parents invest highly in education, $x = 2h$; education expenditure is at its maximum as h is unbounded. This is akin to sending a child to an upper-tier private school (perhaps an elite public school) that charges even greater fees. This offers a significantly better quality of education than a state school, as quality is increasing in h. This also incurs substantially higher costs than an independent school charging only intermediate fees. Nevertheless, this is in return for a

premium education, offering even smaller class sizes, the best teachers and world-class facilities. We could even comment that attending an upper-tier private school will further cultivate the future success of the child (and thus the happiness of the parents) through the greater likelihood of acceptance at the best universities, which will enhance their employment prospects.

The alternative to paying intermediate and high fees is paying little (or zero, in this case) for a free education provided by the government. This is when both parents invest low in the child's human capital; $x = 2l = 0$. This may be of relatively lower quality, but will be a smaller burden on income, allowing parents to derive higher utility from greater disposable income, $y_i - x_i$. It could be argued that a higher household disposable income will make both parents and children better off, through the provision of a stable, comfortable living standard. This means there is initially no clear choice between *h* and *l*.

The sets of investment choice outcomes, rather than the payoffs, are summarised below. This will allow us to interpret the parents' investment behaviour for when θ is greater than and less than 1, under different levels of household labour income, and for varying degrees of *h*.

θ > 1:

$0 < h < \frac{1}{3}$

		Player 2: *W*	Player 2: *H*
Player 1	*W*	*l* , *h*	*l* , *h*
	H	*l* , *h*	*l* , *h*

$\frac{1}{3} < h < 1$

		Player 2: *W*	Player 2: *H*
Player 1	*W*	*h* , *h*	*h* , *h*
	H	*h* , *h*	*h* , *h*

$h > 1$

		Player 2: *W*	Player 2: *H*
Player 1	*W*	*h* , *h*	*h* , *h*
	H	*h* , *h*	*h* , *h*

When $\theta > 1$, player 2 places greater intrinsic value on the total investment in the child's education. This means that player 2 derives relatively more happiness from investing highly in education and from the future success their child will be able to achieve as a consequence. Hence a consistently high investment made by player 2 – regardless of income, or the cost and quality of education – is expected.

In contrast, player 1's selection is influenced by the price and implied quality of the investment. When *h* is low, a private education is similar in cost and quality to a state-provided free education. If this is the case, player 1 invests

low – player 1 recognises it is not worthwhile to invest more (however small) when a similar outcome can be achieved with less (or nil) investment. Player 1 only makes a high investment when the difference in calibre between state and private education is sufficiently large.

θ < 1:

$0 < h < \frac{1}{3}$

		Player 2 W	Player 2 H
Player 1	W	l, l	l, l
	H	l, l	l, l

$\frac{1}{3} < h < 1$

		Player 2 W	Player 2 H
Player 1	W	l, l	l, l
	H	l, l	l, l

$h > 1$

		Player 2 W	Player 2 H
Player 1	W	h, l	h, l
	H	h, l	h, l

When $\theta < 1$, player 2 places relatively less weight on the household's investment in the child's human capital and more on emotional quality. It is therefore unsurprising that player 2 systematically chooses to make a low (under our assumptions, nil) investment, irrespective of income.

Player 1 also consistently invests at a low level for smaller values of h, suggesting as before that player 1 recognises little extra value in a more expensive education. However, player 1 now plays 'Low' for all values of $h < 1$, compared to 'Low' when $h < \frac{1}{3}$. It seems player 1 is more willing to invest when player 2 derives greater happiness from the household's investment. Player 1 only makes a high investment when there is a significantly large price and quality discrepancy between private and state education.

From these observations, we can identify player 2 as more extreme and more welfare-centric than player 1, who is financially shrewd and only prepared to pay for results. Player 1's behaviour is arguably reflected in the payoff function, which is strictly increasing in investment. It is interesting to note that even though θ is absent from player 1's payoff function, its value is correlated with player 1's investment. This could imply some dependence mechanism; an underlying marital influence of player 2 over player 1. When player 2 favours investing either high or low, player 1 is inclined to follow suit. We could also surmise that player 2 has high price elasticity of education, as an explanation for relatively polar outcomes. Player 2's investment is highly sensitive to the

worth they place on the household's total investment, at least relative to player 1.

4.2 Career choices

From the stage 2 NE previously derived, the equilibrium payoffs underlying the investment decisions are displayed below, now as elements of the career decision game Γ_1, for varying values of h and θ.

θ > 1:

$0 < h < \frac{1}{3}$

		Player 2: W	Player 2: H
Player 1	W	$y_1 - l + (h+l)^2 + \frac{y_1 + y_2}{2}$, $y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - l + (h+l)^2 + \frac{y_1 + t_2}{2}$, $-h + \theta(h+l) + \left(\frac{y_1 + t_2}{2}\right)^2$
	H	$-l + (h+l)^2 + \frac{y_2}{2}$, $y_2 - h + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$	$-l + (h+l)^2 + \frac{t_2}{2}$, $-h + \theta(h+l) + \left(\frac{t_2}{2}\right)^2$

$\frac{1}{3} < h < 1$

		Player 2: W	Player 2: H
Player 1	W	$y_1 - h + 4h^2 + \frac{y_1 + y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - h + 4h^2 + \frac{y_1 + t_2}{2}$, $-h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2$
	H	$-h + 4h^2 + \frac{y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2$	$-h + 4h^2 + \frac{t_2}{2}$, $-h + 2\theta h + \left(\frac{t_2}{2}\right)^2$

$h > 1$

Player 1 \ Player 2	W	H
W	$y_1 - h + 4h^2 + \frac{y_1 + y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - h + 4h^2 + \frac{y_1 + t_2}{2}$, $-h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2$
H	$-h + 4h^2 + \frac{y_2}{2}$, $y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2$	$-h + 4h^2 + \frac{t_2}{2}$, $-h + 2\theta h + \left(\frac{t_2}{2}\right)^2$

$\theta < 1$:

$0 < h < \frac{1}{3}$

Player 1 \ Player 2	W	H
W	$y_1 - l + 4l^2 + \frac{y_1 + y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - l + 4l^2 + \frac{y_1 + t_2}{2}$, $-l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2$
H	$-l + 4l^2 + \frac{y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2$	$-l + 4l^2 + \frac{t_2}{2}$, $-l + 2\theta l + \left(\frac{t_2}{2}\right)^2$

$\frac{1}{3} < h < 1$

Player 1 \ Player 2	W	H
W	$y_1 - l + 4l^2 + \frac{y_1 + y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - l + 4l^2 + \frac{y_1 + t_2}{2}$, $-l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2$
H	$-l + 4l^2 + \frac{y_2}{2}$, $y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2$	$-l + 4l^2 + \frac{t_2}{2}$, $-l + 2\theta l + \left(\frac{t_2}{2}\right)^2$

$h > 1$

		Player 2: W	Player 2: H
Player 1	W	$y_1 - h + (h+l)^2 + \frac{y_1 + y_2}{2}$, $y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$	$y_1 - h + (h+l)^2 + \frac{y_1 + t_2}{2}$, $-l + \theta(h+l) + \left(\frac{y_1 + t_2}{2}\right)^2$
	H	$-h + (h+l)^2 + \frac{y_2}{2}$, $y_2 - l + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$	$-h + (h+l)^2 + \frac{t_2}{2}$, $-l + \theta(h+l) + \left(\frac{t_2}{2}\right)^2$

Theorem 4: *{W,W} is a NE, for all h* $\notin \{\frac{1}{3}, 1\}$*, though it is not unique.*

To establish the above theorem, we use an accompanying lemma.

Lemma 1. *Assuming income is positive, {W,W}* and *{W,H} are NE.*

Proof.
Consider the general form of Γ_1.

Γ_1

		Player 2: W	Player 2: H
Player 1	W	I , J	K , L
	H	M , N	O , P

From the previously defined payoffs, observe that in every scenario, working *(W)* is a strictly dominant strategy for player 1 and, by symmetry, staying at home *(H)* is always strictly dominated. Payoff I is always greater than payoff M and payoff K is always greater than payoff O for non-negative incomes.

For player 2, the result is not so clear. Suppose the game is solved by strict dominance; we firstly eliminate player 1's dominated *(H)* strategy.[5]

Γ_1

		Player 2: W	Player 2: H
Player 1	W	I , J	K , L

For all scenarios, player 2's payoffs simplify to the following.

[5] For strictly dominated strategies, the order of elimination is immaterial.

If *{W,W}* is a unique NE, J > L: $$y_2 + \left(\frac{y_1 + y_2}{2}\right)^2 > \left(\frac{y_1 + t_2}{2}\right)^2$$

If *{W,H}* is a unique NE, L > J: $$y_2 + \left(\frac{y_1 + y_2}{2}\right)^2 < \left(\frac{y_1 + t_2}{2}\right)^2$$

If *{W,W}* and *{W,H}* are NE, L = J: [6] $$y_2 + \left(\frac{y_1 + y_2}{2}\right)^2 = \left(\frac{y_1 + t_2}{2}\right)^2$$

If we make the blanket assumption that $y_2 > t_2$, then *{W,W}* will be the unique NE of Γ_1. This is conceivable. Player 2 would draw greater utility from the better lifestyle associated with additional household income and the improvement in their child's development attributed to a higher standard of living than the utility derived from the contribution to said development that time spent at home could make. Unless player 2's preferences were exceptionally altruistic towards the emotional quality of the child, this assumption would hold.

However, should an agent have preferences such that they believe time spent at home nurturing their children would bring greater benefit than money could to both the child's and their own welfare, it may be that $t_2 > y_2$. In this instance, player 2 would have incentive to deviate from *(W)* and *{W,H}* could be reached as a unique pure strategy NE. In these circumstances, it would be optimal for player 1 to work and earn the income and education investment they so highly value, whilst player 2, our child-carer, stays at home. This event would support the aforementioned suggestion of player 2 as a female spouse.

{H,H} and *{H,W}* could never be reached as pure strategy NE as they would require individual income to be negative. This is not feasible in this model unless we introduced credit as a means of supplementing income. It is irrelevant to this investigation, so non-negative income is strictly maintained.

4.3 Optimal outcomes

{W,W} and *{W,H}* are the possible pure strategy NE in each possible Γ_1 sub-game. We now investigate under which conditions players' utility is maximised given these career outcomes and our conditional values for h and θ in order to ascertain the optimal paths of our players.

[6] Assuming player 2 chooses randomly when indifferent.

{W,W}:

Comparing payoffs for both players at *{W,W}* under all six conditions:

For both players, when $\theta > 1$ and $h > 1$, utility is highest. Players can achieve the highest possible happiness when both spouses invest highly in private education, and this education is of superior quality to state-provided education. We can interpret this as our ideal situation; parents are presented with a choice of school types and maximise the future success of their child by providing them with the best possible education.

For both players, when $\theta < 1$ and $0 < h < \frac{1}{3}$, utility is lowest. Both choose to send children to state schools in the knowledge that there is little to be gained from investing any higher. This is the worst possible scenario if both parents work. The income earned from working is arguably best spent on providing offspring with a quality education. However, in this situation, the range of available schooling is limited, thus a high investment would be wasted. Both parents work, therefore no time is spent at home by either parent. In the context of this model, this is player 2's prerogative, thus what the child lacks in education provision is exacerbated by deficient emotional support.

{W,H}:

Comparing payoffs for both players at *{W,H}* under all six conditions:

As before for both players, when $\theta > 1$, and $h > 1$, utility is maximised. Players receive the highest possible payoff when both players have high expenditure on human capital, and this education is of significantly greater quality relative to a free state-provided education. However, in this case, player 2 now stays at home and does not earn any income. If we interpret player 2 as female, this is consistent with Browning (1992). Although Browning struggles to define an exact relationship, he associates the presence of young children in a household with lower labour supplied by the mother. Under the assumptions that allow *{W,H}* to be a unique NE, player 2 has preferences that display exceptional relative altruism towards the emotional quality of the child (Becker, 1981) thus opts to stay at home to cultivate their development.

The worst attainable outcome is as before: when $\theta < 1$ and $0 < h < \frac{1}{3}$. The result is not so dire. Player 2 is now at home raising the child – what is lacking in education is counteracted by the presence of a caring parent.

It is of critical importance to remember that θ and h are determined exogenously. Players have no control over their decision-making environment and so are susceptible to all *{W,W}* and *{W,H}* outcomes. We compare the best and worst scenarios simply to evaluate.

5. A cooperative extension

We now deviate from the simultaneous decision-making model, where both agents are equally weighted, and explore an alternative type of household; a cooperative environment, where parents communicate.

In cooperative games, players come together in different coalitions and make enforceable contracts. This is in contrast to non-cooperative games, where players make decisions independently and the outcomes are self-enforcing. We are now concerned with what our theoretical couple can jointly achieve, rather than the decisions they can individually make.

In cooperative equilibrium, players 1 and 2 behave as a coalition, and maximise the sum of their utilities. In this game, we consider the grand coalition, $S = \{1,2\}$. Rather than selfishly maximising their own happiness, it is now in the players' interest to jointly maximise the welfare of the entire household, $v(S) = U_1 + U_2$, by choosing the household's total investment, $x = x_1 + x_2$.

UMP:

$$\max_{x=x_1+x_2} v(S) = y_1 + y_2 - x - x^2 + \theta x + \left(\frac{y_1 + y_2 + t_2}{2}\right) + \left(\frac{y_1 + y_2 + t_2}{2}\right)^2$$

$$\frac{\partial v(S)}{\partial x} = -1 + 2x + \theta = 0 \quad , \quad \frac{\partial^2 v(S)}{dx^2} = 2 > 0$$

$$\Rightarrow x = \frac{1-\theta}{2}$$

Observe that this would be the result for all career outcomes, whether either or both players worked and earned an income, forgoing time spent at home. Also, we have assumed equality of power of spouses.

If $\theta > 1$, x is negative. Given our earlier assumptions, this cannot be (x is non-negative), therefore it must be that $x = 0$. This corresponds with our low investment outcome, *(l, l)*. The positive second order derivative signals this is a minimum. As high investment (*h*) is unbounded, the maximum can be deduced as:

$$x^* = y_1 + y_2$$

$$\Leftrightarrow x^* = (h,h) = 2h$$

Cooperative equilibrium is therefore where both players invest highly in their child's human capital, regardless of household income. This is a pleasant result. We can interpret the cooperative game as representative of a united couple, seeking to make their household as well off as possible together by maximising the output potential of their child. This is arguably a more realistic scenario than what has been previously assumed; selfish spouses who disregard their partner's happiness.

If $\theta < 1$, x is positive. The outcome is still a minimum, and therefore at maximum, unbounded h still holds. However, there is a difference. Previously, the minimum was at zero (nil investment). The minimum is now a positive value, implying a smaller discrepancy in the perceived quality between state and private education. This is consistent with our interpretation of θ; when $\theta < 1$, player 2 values education less and sees lower worth in an expensive private education relative to a free state education.

To summarise, in a cooperative environment, parents will consistently spend highly to provide their child with a quality education. Although a warming result, this behaviour could be interpreted as irrational. If household income is low (if neither parent earns an income), investing highly could make both the parents and the child worse off. Spending beyond a household's means could erode living standards to a point below a basic sustenance. In order to eliminate this possibility, we would need to impose an upper bound on the value of h, such that h cannot absorb a fraction of income greater than that which would leave the household with less than a moderate quality of life with all basic amenities. Any expenditure beyond this would be nonsensical and would leave us with a meaningless result.

6. Conclusion

This paper has developed a two-stage non-cooperative household game, where parents decide upon their careers and individual investments into their child's human capital. The model is solved by backward induction for sub-game perfect NE to find the optimal paths of both parents. The model is then extended for a cooperative equilibrium.

In the non-cooperative environment under pure strategies, both parents working is a NE, though there are alternative outcomes when the conditions underlying the decision are varied. The investment behaviour of agents is critically affected by the magnitude of the cost and quality of a high-cost education relative to a low-cost education, and the extent of the intrinsic value that player 2 (our implied female spouse) places on education as a driver of the child's future success. Agents are not deterred by high schooling fees as long as the price is a reflection of quality. Where a similar standard is attainable by free education, parents are more likely to opt for the low-cost alternative.

The cooperative outcome compliments this somewhat; parents will always seek to maximise the welfare of their child by providing an education of the highest possible quality, irrespective of their income. However, the boundless

outcome is troubling and accentuates the need for further development to derive a robust theory.

A simplistic model has been constructed and some key issues explored, which provide a platform to consider other issues. To truly reflect marriage, we require greater interdependence between players. This has been proved intermittently in the empirical literature, and would allow us to demonstrate the internal dynamics of a household. Also, another tier to the game could be introduced. For an example, earlier stages where the parents of our agents make career and education investment decisions that alter their preferences.

Roles of men and women in the household have been hinted in this paper but there is scope for further investigation. The preferences of agents could be experimented with to reflect changing attitudes and perceived stereotypes of men and women within and across the world.

We have not considered the possibility of a wage differential between male and female spouses. Empirical evidence suggests a significant and robust gender-wage gap, such that women earn less than men ceteris paribus. If we view player 1 as male and player 2 as female and impose this differential, perhaps women would have less incentive to work for lower rewards. They would be more inclined to stay at home and maximise their time investment in the child's welfare rather than in education.

No assumptions have been made over the preferences of players for work, leisure and home; we have only accepted that work generates income and prevents agents caring for children. Introducing further preferences would surely affect the career decisions. Following this, we could introduce a trade-off between x and e; the future output of the child versus their emotional quality. There has been no clarification of which is more important, or the possibility and effect of a bi-causal relationship.

There is a wealth of interesting comparisons to be made between developed and developing countries using this model.

Developing country governments often lack sufficient funds to provide widespread free state schools. It is therefore relatively common for parents in developing countries to establish independent schools and pay for their children's education (Hillman and Jenkner, 2003). As household incomes in developing countries are often much lower than their developed counterparts, it would be interesting to test for a potential trade-off between allocating income to education and the expenditure required to improve standards of living.

It would be interesting to observe if the behaviour of parents changed should they have multiple children. Although fertility rates in developing countries

are declining, they are still considerably higher than in developed countries[7]. We have theorised that parents have a propensity to invest highly in a quality education for a child. It would be interesting to see whether this holds for more than one child, and in what functional form education expenditure per child increases or decreases. If we assumed heterogeneous children differing in their future earning capabilities, perhaps parents would invest more in a child with greater potential. Instead, they may compensate a less-able child with a better education to eliminate the inequality between children.

The range of unexplored issues highlights this paper as a stepping-stone for further investigation.

[7] *Source:* World Bank World Development Indicators (2010).

Appendix

3. Analysis

Theorem 1 and 2

Testing of all possible NE *(h,h) (h,l), (l,h), (l,l)* under all **C** conditions is demonstrated, assuming $l = 0$.

For an investment combination (x_1, x_2) to be a NE, the payoffs must be such that neither player has incentive to deviate.

WW:

		Player 2: h	Player 2: l
Player 1	h	$A: y_1 - h + 4h^2 + \frac{y_1 + y_2}{2}$, $B: y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2$	$C: y_1 - h + (h+l)^2 + \frac{y_1 + y_2}{2}$, $D: y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$
	l	$E: y_1 - l + (h+l)^2 + \frac{y_1 + y_2}{2}$, $F: y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$	$G: y_1 - l + 4l^2 + \frac{y_1 + y_2}{2}$, $H: y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2$

(h,h) For *(h,h)*, to be a NE, A > E and B > D.

A > E

$$y_1 - h + 4h^2 + \frac{y_1 + y_2}{2} > y_1 - l + (h+l)^2 + \frac{y_1 + y_2}{2}$$
$$\Rightarrow -h + 4h^2 > h^2$$
$$\Rightarrow 3h^2 > h$$
$$\Rightarrow h > \tfrac{1}{3}$$

B > D

$$y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2 > y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$$
$$\Rightarrow -h + 2\theta h > \theta h$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(h,l) For *(h,l)* to be a NE, C > G and D > B.

C > G

$$y_1 - h + (h+l)^2 + \frac{y_1 + y_2}{2} > y_1 - l + 4l^2 + \frac{y_1 + y_2}{2}$$
$$\Rightarrow -h + h^2 > 0$$
$$\Rightarrow h^2 > h$$
$$\Rightarrow h > 1$$

D > B

$$y_2 - l + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2 > y_2 - h + 2\theta h + \left(\frac{y_1 + y_2}{2}\right)^2$$
$$\Rightarrow \theta h > -h + 2\theta h$$
$$\Rightarrow h > \theta h$$
$$\Rightarrow 1 > \theta$$

(l,h) For *(l,h)* to be a NE, E > A and F > H.

E > A
$$y_1 - l + (h+l)^2 + \frac{y_1 + y_2}{2} > y_1 - h + 4h^2 + \frac{y_1 + y_2}{2}$$
$$\Rightarrow h^2 > -h + 4h^2$$
$$\Rightarrow h > 3h^2$$
$$\Rightarrow \tfrac{1}{3} > h$$

F > H
$$y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2 > y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2$$
$$\Rightarrow -h + \theta h > 0$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(l,l) For *(l,l)* to be a NE, G > C and H > F.

G > C
$$y_1 - l + 4l^2 + \frac{y_1 + y_2}{2} > y_1 - h + (h+l)^2 + \frac{y_1 + y_2}{2}$$
$$\Rightarrow 0 > -h + h^2$$
$$\Rightarrow h > h^2$$
$$\Rightarrow 1 > h$$

H > F
$$y_2 - l + 2\theta l + \left(\frac{y_1 + y_2}{2}\right)^2 > y_2 - h + \theta(h+l) + \left(\frac{y_1 + y_2}{2}\right)^2$$
$$\Rightarrow 0 > -h + \theta h$$
$$\Rightarrow h > \theta h$$
$$\Rightarrow 1 > \theta$$

WH:

Player 2

		h	l
Player 1	h	$A: y_1 - h + 4h^2 + \frac{y_1 + t_2}{2}$, $B: -h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2$	$C: y_1 - h + (h+l)^2 + \frac{y_1 + t_2}{2}$, $D: -l + \theta(h+l) + \left(\frac{y_1 + t_2}{2}\right)^2$
	l	$E: y_1 - l + (h+l)^2 + \frac{y_1 + t_2}{2}$, $F: -h + \theta(h+l) + \left(\frac{y_1 + t_2}{2}\right)^2$	$G: y_1 - l + 4l^2 + \frac{y_1 + t_2}{2}$, $H: -l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2$

(h,h) For *(h,h)* to be a NE, A > E and B > D.

A > E
$$y_1 - h + 4h^2 + \frac{y_1 + t_2}{2} > y_1 - l + (h+l)^2 + \frac{y_1 + t_2}{2}$$
$$\Rightarrow -h + 4h^2 > h^2$$
$$\Rightarrow 3h^2 > h$$
$$\Rightarrow h > \tfrac{1}{3}$$

B > D

$$-h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2 > -l + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2$$
$$\Rightarrow -h + 2\theta h > \theta h$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(h,l) For *(h,l)* to be a NE, C > G and D > B.

C > G

$$y_1 - h + (h + l)^2 + \frac{y_1 + t_2}{2} > y_1 - l + 4l^2 + \frac{y_1 + t_2}{2}$$
$$\Rightarrow -h + h^2 > 0$$
$$\Rightarrow h^2 > h$$
$$\Rightarrow h > 1$$

D > B

$$-l + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2 > -h + 2\theta h + \left(\frac{y_1 + t_2}{2}\right)^2$$
$$\Rightarrow \theta h > -h + 2\theta h$$
$$\Rightarrow h > \theta h$$
$$\Rightarrow 1 > \theta$$

(l,h) For *(l,h)* to be a NE, E > A and F > H.

E > A

$$y_1 - l + (h + l)^2 + \frac{y_1 + t_2}{2} > y_1 - h + 4h^2 + \frac{y_1 + t_2}{2}$$
$$\Rightarrow h^2 > -h + 4h^2$$
$$\Rightarrow h > 3h^2$$
$$\Rightarrow \tfrac{1}{3} > h$$

F > H

$$-h + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2 > -l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2$$
$$\Rightarrow -h + \theta h > 0$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(l,l) For *(l,l)* to be a NE, G > C and H > F.

G > C

$$y_1 - l + 4l^2 + \frac{y_1 + t_2}{2} > y_1 - h + (h + l)^2 + \frac{y_1 + t_2}{2}$$
$$\Rightarrow 0 > -h + h^2$$
$$\Rightarrow h > h^2$$
$$\Rightarrow 1 > h$$

H > F

$$-l + 2\theta l + \left(\frac{y_1 + t_2}{2}\right)^2 > -h + \theta(h + l) + \left(\frac{y_1 + t_2}{2}\right)^2$$
$$\Rightarrow 0 > -h + \theta h$$
$$\Rightarrow h > \theta h$$
$$\Rightarrow 1 > \theta$$

HW:

	Player 2	
	h	*l*

Player 1			
	h	$A: -h + 4h^2 + \frac{y_2}{2}$, $B: y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2$	$C: -h + (h+l)^2 + \frac{y_2}{2}$, $D: y_2 - l + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$
	l	$E: -l + (h+l)^2 + \frac{y_2}{2}$, $F: y_2 - h + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$	$G: -l + 4l^2 + \frac{y_2}{2}$, $H: y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2$

(h,h) For *(h,h)* to be a NE, A > E and B > D.

A > E

$$-h + 4h^2 + \frac{y_2}{2} > -l + (h+l)^2 + \frac{y_2}{2}$$
$$\Rightarrow -h + 4h^2 > h^2$$
$$\Rightarrow 3h^2 > h$$
$$\Rightarrow h > \tfrac{1}{3}$$

B > D

$$y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2 > y_2 - l + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$$
$$\Rightarrow -h + 2\theta h > \theta h$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(h,l) For *(h,l)* to be a NE, C > G and D > B.

C > G

$$-h + (h+l)^2 + \frac{y_2}{2} > -l + 4l^2 + \frac{y_2}{2}$$
$$\Rightarrow -h + h^2 > 0$$
$$\Rightarrow h^2 > h$$
$$\Rightarrow h > 1$$

D > B

$$y_2 - l + \theta(h+l) + \left(\frac{y_2}{2}\right)^2 > y_2 - h + 2\theta h + \left(\frac{y_2}{2}\right)^2$$
$$\Rightarrow \theta h > -h + 2\theta h$$
$$\Rightarrow h > \theta h$$
$$\Rightarrow 1 > \theta$$

(l,h) For *(l,h)* to be a NE, E > A and F > H.

E > A

$$-l + (h+l)^2 + \frac{y_2}{2} > -h + 4h^2 + \frac{y_2}{2}$$
$$\Rightarrow h^2 > -h + 4h^2$$
$$\Rightarrow h > 3h^2$$
$$\Rightarrow \tfrac{1}{3} > h$$

F > H

$$y_2 - h + \theta(h+l) + \left(\frac{y_2}{2}\right)^2 > y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2$$
$$\Rightarrow -h + \theta h > 0$$
$$\Rightarrow \theta h > h$$
$$\Rightarrow \theta > 1$$

(l,l) For *(l,l)* to be a NE, G > C and H > F.

G > C $\quad -l + 4l^2 + \frac{y_2}{2} > -h + (h+l)^2 + \frac{y_2}{2}$

$\Rightarrow 0 > -h + h^2$

$\Rightarrow h > h^2$

$\Rightarrow 1 > h$

H > F $\quad y_2 - l + 2\theta l + \left(\frac{y_2}{2}\right)^2 > y_2 - h + \theta(h+l) + \left(\frac{y_2}{2}\right)^2$

$\Rightarrow 0 > -h + \theta h$

$\Rightarrow h > \theta h$

$\Rightarrow 1 > \theta$

HH

		Player 2	
		h	l
Player 1	h	$A: -h + 4h^2 + \frac{t_2}{2}$, $B: -h + 2\theta h + \left(\frac{t_2}{2}\right)^2$	$C: -h + (h+l)^2 + \frac{t_2}{2}$, $D: -l + \theta(h+l) + \left(\frac{t_2}{2}\right)^2$
	l	$E: -l + (h+l)^2 + \frac{t_2}{2}$, $F: -h + \theta(h+l) + \left(\frac{t_2}{2}\right)^2$	$G: -l + 4l^2 + \frac{t_2}{2}$, $H: -l + 2\theta l + \left(\frac{t_2}{2}\right)^2$

(h,h) For *(h,h)* to be a NE, A > E and B > D.

A > E $\quad -h + 4h^2 + \frac{t_2}{2} > -l + (h+l)^2 + \frac{t_2}{2}$

$\Rightarrow -h + 4h^2 > h^2$

$\Rightarrow 3h^2 > h$

$\Rightarrow h > \frac{1}{3}$

B > D $\quad -h + 2\theta h + \left(\frac{t_2}{2}\right)^2 > -l + \theta(h+l) + \left(\frac{t_2}{2}\right)^2$

$\Rightarrow -h + 2\theta h > \theta h$

$\Rightarrow \theta h > h$

$\Rightarrow \theta > 1$

(h,l) For *(h,l)* to be a NE, C > G and D > B.

C > G $\quad -h + (h+l)^2 + \frac{t_2}{2} > -l + 4l^2 + \frac{t_2}{2}$

$\Rightarrow -h + h^2 > 0$

$\Rightarrow h^2 > h$

$\Rightarrow h > 1$

D > B $\quad -l+\theta(h+l)+\left(\frac{t_2}{2}\right)^2 > -h+2\theta h+\left(\frac{t_2}{2}\right)^2$

$\Rightarrow \theta h > -h+2\theta h$

$\Rightarrow h > \theta h$

$\Rightarrow 1 > \theta$

(l,h) For *(l,h)* to be a NE, E > A and F > H.

E > A $\quad -l+(h+l)^2+\frac{t_2}{2} > -h+4h^2+\frac{t_2}{2}$

$\Rightarrow h^2 > -h+4h^2$

$\Rightarrow h > 3h^2$

$\Rightarrow \frac{1}{3} > h$

F > H $\quad -h+\theta(h+l)+\left(\frac{t_2}{2}\right)^2 > -l+2\theta l+\left(\frac{t_2}{2}\right)^2$

$\Rightarrow -h+\theta h > 0$

$\Rightarrow \theta h > h$

$\Rightarrow \theta > 1$

(l,l) For *(l,l)* to be a NE, G > C and H > F.

G > C $\quad -l+4l^2+\frac{t_2}{2} > -h+(h+l)^2+\frac{t_2}{2}$

$\Rightarrow 0 > -h+h^2$

$\Rightarrow h > h^2$

$\Rightarrow 1 > h$

H > F $\quad -l+2\theta l+\left(\frac{t_2}{2}\right)^2 > -h+\theta(h+l)+\left(\frac{t_2}{2}\right)^2$

$\Rightarrow 0 > -h+\theta h$

$\Rightarrow h > \theta h$

$\Rightarrow 1 > \theta$

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