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Stock Index Pricing with Random Walk and Agent-based Models

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Abstract

The objective of this work is to empirically test the EMH and compare its results to those of a viable competitor using computational simulation. Specifically, the individual-agent approach has been gaining momentum recently as the appropriate numerical tools are now widely available (Bonaneau 2002; Cioffi-Revilla 2002; Diks et al. 2007; Gilbert, Bankes 2002; Inchiosa, Parker 2002; Tesfatsion 2002). This fact, coupled with intensifying doubts concerning the validity of efficient-markets theory, has led to intensive use of the agent-based approach with computational agent-based modeling (ABM) of financial markets (Bonabeau 2002). Although multiple theories currently compete with the EMH to varying degrees, we focus explicitly on the use of ABM to generate results consistent with Hang Seng and Nikkei 225 price changes.

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Stock Index Pricing with Random Walk and Agent-based Models

Scott Swisher

I. Introduction[1]

"The image one gets from the news is that financial markets are dominated by *people*. In contrast, a reading of a standard finance textbook … can create the impression that financial markets are nearly devoid of human activity" (Thaler 1993). The field of asset pricing, specifically the valuation of stock market shares, has historically played host to a number of contradictory theories regarding the determination of prices. As the debate currently stands, the efficient market hypothesis (EMH) has assumed a dominant position following the enumeration of rational expectations theory at the University of Chicago (primarily) in the 1960s by Muth, Fama, and Lucas (Sheffrin 1996; Shiller 2000). Fama's specification is that "security prices always fully reflect the available information" in an efficient market (Shleifer 2000). Initial econometric testing regarding the efficient-markets theory confirmed germane hypotheses, but by the mid-1970s academics were increasingly skeptical due to the restrictive nature of the assumptions and contradictory empirical findings (Sheffrin 1996). As a result, alternative theories involving non-rational actors were developed under the banner of behavioral finance by Shiller, De Bondt, Thaler, Roll, and others; however, the EMH remained the *de facto* central paradigm of finance, a position it has held for over thirty years (Hirshleifer 2001; Sheffrin 1996; Shleifer 2000).

In such a context, questioning the current theory *vis-à-vis* well-developed alternatives is perfectly reasonable because the consensus is not well-defined (Arthur et al. 1997; Baker, Wurgler 2007; De Bondt, Thaler 1984; Hirshleifer 2001; Hong, Stein 2007; Shleifer 2000; Worthington, Higgs 2003). The objective of this work is to empirically test the EMH and compare its results to those of a viable competitor using computational simulation. Specifically, the individual-agent approach has been gaining momentum recently as the appropriate numerical tools are now widely available (Bonaneau 2002; Cioffi-Revilla 2002; Diks et al. 2007; Gilbert, Bankes 2002; Inchiosa, Parker 2002; Tesfatsion 2002). This fact, coupled with intensifying doubts concerning the validity of efficient-markets theory, has led to intensive use of the agent-based approach with computational agent-based modeling (ABM) of financial markets (Bonabeau 2002). Although multiple theories currently compete with the EMH to varying degrees, we focus explicitly on the use of ABM to generate results consistent with Hang Seng and Nikkei 225 price changes. The agent-based results are compared with output from a random-walk model directly inspired by the tenets of the EMH;

1 I would like to thank the members of my honors research committee at Illinois Wesleyan University: T.X. He, S.H. Lee, N. Jaggi, and M. Seeborg. I. Odinaka, an undergraduate at the university, gave me much-needed advice regarding Mathematica programming.

model parameters are selected such that each model is run with reduced error. Stationarity[2] is used as a comparative metric in order to assess model accuracy and appropriate characterization of historical data. Our results imply that the random-walk model is more consistent with the empirical facts in this particular situation.

Throughout the 1990s, a perception has been developing that efficient-markets theory is inconsistent with the available data; critics cite price volatility in excess of what would be dictated by changes in fundamental value as evidence (Thaler 1993; Shleifer 2000). Additionally, those in behavioral finance argue that no time-constrained individual could ever possess the computing power required to calculate and recalculate the fundamental value of all stocks in a diversified portfolio (Hirshleifer 2001). Such skepticism is countered by those empirical results that do confirm the EMH (Pearce, Roley 1985); efficient financial markets are consistent with laissez-faire and the innate wisdom of unconstrained market forces (Ormerod 1998). The net result is a field characterized by theoretical conflict between alternative theories, a situation not uncommon in the economics discipline, but the dispute is as much dogmatic and political as it is empirical and scientific (Schleifer 2000). Stock markets have been traditionally viewed as the apogee of free-market idealism; shares are traded on a daily basis without significant restriction, so each stock price should represent actual (fundamental) value. Deviation from the correct valuation is rapidly purged from the market system by the broad mass of fundamentalist traders; as such, the EMH posits that each stock price reflects the discounted present value of the sum of future earnings. Therefore, re-examining the mechanism that determines stock prices can be viewed as an attack upon the most important foundations of market efficiency.

Confirming the applicability of bounded rationality and the imperfection of market traders leads to a reexamination of previous bubble-corrective incidents that brought financial ruin to millions and persistent negative economic consequences (Shiller 2000). Economists are prompted to question if the market can be manipulated to make mistakes as speculators push asset prices higher for their own self-enrichment at the cost of macroeconomic stability (Raines, Leathers 2000). Regardless of the evidence, the efficient-markets theory has an incumbent advantage that can be nearly impossible to nullify. Resistance to theoretical change is also due to the esoteric nature of the topic, and although data availability is no longer a problem (Pearce 1984), financial data require statistical analysis using complex econometric modeling (Worthington, Higgs 2003). The utility of the ARCH model in performing data analysis in finance, for example, is due to the heteroscedastic and autoregressive properties of stock prices over time; the time-series are characterized by short-lived bursts of volatility (Shumway, Stoffer 2006). Extensive data availability can also be viewed as a mixed blessing: although ample series are available for investigation, data mining (i.e. selecting the data set that maximizes model performance) can become endemic to the study of random walk models (Hirshleifer 2001). Consequently, rejection or acceptance of the EMH is a function of the data set used, so no generalized conclusions are drawn by the discipline.

Our objective here is to empirically test the EMH and compare its results to those of an agent-based alternative using Mathematica-based computational simulation. The two models are empirically compared using the criteria of stationarity and autoregressive behavior. The agent-based approach used, termed the "ant trader" model, is based on the ant model established by Kirman in his 1993 work "Ants, Rationality, and Recruitment". Daily returns of the Hang Seng and Nikkei 225 indices are used over the periods 1987-2007 and 1984-2007, respectively. The model that most accurately reflects the conditions present in these real-world markets in terms of stationarity will be theoretically preferred.

2 A set of time-series data is said to be stationary if its mean and variance do not change as a function of time.

This study is not immune to the issue of limited applicability of results, and the determinations made here regarding the efficient-markets theory are not necessarily extensible to other indices that differ non-trivially in terms of period under consideration, composition of stocks, industrial concentration, or regional factors (Worthington, Higgs 2003; Huber 1995). Disagreement between the EMH and its opponents will continue regardless of new scholarly publications because stock market efficiency is a function of index characteristics and time; some indices are more adept at incorporating information than others[3] (Worthington, Higgs 2003).

II. Review of Literature

Arguments made for efficient markets were originally theoretical, consisting of the formation and application of rational expectations by Muth, Lucas, and so on; the idea of rational expectations quickly migrated from macroeconomics to finance, resulting in the random-walk model of stock prices. The EMH is appealing partly because of its implications: stock market prices reflect all available information concerning the discounted expected value of future corporate earnings streams, i.e. the capital asset pricing model, or CAPM. Consequently, no long-term profitable trading rule can be established as stock prices engage in a random walk around the fundamental value. The theory only provides a fundamentalist trading rule as follows: sell if the price is above the "true" value (overvalued) and buy/hold if price is below the "true" value (undervalued), where the true value reflects the future earnings stream of the asset (Sheffrin 1996). Efficient-markets theory implies that financial analysts are redundant when all market actors have access to complete information, and technical analysis based on short-run trends is ineffectual (Hirshleifer 2001; Shleifer 2000). Volatile day trading is useless when the underlying true value is not changing, so all the rational investor has to do is buy and hold undervalued[4] stocks until they eventually become overvalued in the course of a random walk stochastic process. The discovery of a long-term profitable trading rule would invalidate the efficient-markets theory, however (Thaler 1993).

The EMH can be broken down into three subclasses as defined by Fama in his seminal 1970 work "Efficient Capital Markets: A Review of Theory and Empirical Work": a particular market may exhibit weak, semi-strong, or strong efficiency. A market is said to be weakly efficient if complete awareness of past information does not improve long-run portfolio profitability. Semi-strong efficiency is satisfied if portfolio return cannot be increased using knowledge of publically-available information (Shleifer 2000). The strict criterion of strong-form efficiency is the most difficult to prove; we say that a market is strongly efficient if even insider (non-public) information cannot improve portfolio return. As such, the majority of scholars deal with weak-form efficiency in financial markets because it is difficult to properly treat the insider information set econometrically[5] (Sheffrin 1996). Although this paper does not directly test for market efficiency, the definitions are worth noting due to their importance in the EMH framework. The random-walk specification that will be used is weakly efficient; a more stringent specification would require additional evidence regarding the information set, which is beyond the scope of this paper.

A voluminous literature has grown around the efficient market hypothesis; a concise summary is provided by Sheffrin in Rational Expectations (1996) as cited previously. The EMH assumes the following: investors are rational actors,

3 Co-existence of efficient and inefficient markets implies that both sides can find supporting evidence, which leads to contradictory results.

4 Undervalued, i.e. below fundamental value, assuming that fundamental value is well-defined and can be computed.

5 By definition, insider information is not known to the public; therefore, compiling data on such a topic might prove impossible. Individuals with insider information will not want to divulge the extent of their knowledge due to fear of prosecution or loss of trading advantage.

imperfectly rational investors trade randomly with zero net effect, and arbitrage undertaken by rational actors nullifies the actions of non-rational traders (Shleifer 2000). Of these three, the most important assumption is arbitrage; if of sufficient number, rational arbitrageurs[6] can effectively purge the market of its irrational elements through fundamentals-based trading. Therefore, the requirement that market agents are perfectly rational can be relaxed and the theory remains consistent under suboptimal (i.e. realistic) conditions. Under these assumptions with a market composed of risk-neutral rational investors, mathematical economists Samuelson and Mandelbrot proved that returns follow a random walk process in the mid-1970s (Shleifer 2000). At this time, empirical evidence overwhelmingly supported the efficient-markets theory and arbitrage was able to explain away isolated outbreaks of irrational "noise trader" behavior. The 1972 event study of Scholes suggested that arbitrageurs can only operate when near-perfect substitutes are available for an individual stock, but his work generally confirmed the EMH regardless of the prerequisites. Of note here is that the EMH relies upon a multitude of powerful assumptions, mainly the primacy of rational, fundamentalist traders. The entire logical argument is invalidated with a violated assumption, but empirical evidence is also supportive.

Empirical testing has proven effective in validating the efficient-markets hypothesis; see Pearce and Roley "Stock Prices and Economic News" and their subsequent confirmation of the EMH (1985). The authors use S&P500 return data coupled with data sources that address expectations and announcements; expectations reflect the state of the information set, while announcements stochastically shock the information set. Theoretically, they adapt the rational expectations framework to the question of predicting changes in stock index prices as follows:

$$\Delta SP_t = a + bx_t^u + cx_t^e + d_i \sum_{i=1}^{m} x_{t-i}^u + e_t \qquad (2.1)$$

The change in stock price at time t is a function of the unexpected announcements vector $\mathbf{x}^u$, expected announcements vector $\mathbf{x}^e$, all previous unexpected news $\sum \mathbf{x}^u$, and an error term e indexed by t. Coefficient *b* should be significantly non-zero, while coefficients *c* and *d* are predicted to be zero in accordance with the EMH. This is because only newly-presented unexpected news should serve as a stochastic shock; expected news and previous surprises ought to be integrated into the price already.

Pearce and Roley find that unexpected announcements induce nearly-instantaneous changes in the index price, but expected (anticipated) announcements do not have a statistically significant effect on stock prices (1985). These results concur with the theoretical predictions of the EMH; only surprise changes in the information set lead to non-trivial stock price movements. Therefore, stock prices reflect all available information, which includes expectations about future announcements regarding monetary policy and corporate finance. Pearce and Roley used the efficient-markets theory as their null hypothesis when conducting statistical tests regarding regression coefficients, so more precisely the authors did not disprove the theory. Their paper is representative of an extensive body of work that has failed to reject the EMH, insofar as a failure to reject represents validation and lends credibility to the proponents of rational expectations.

The random walk model has been specified in a number of increasingly sophisticated ways as per Hagerman and Richmond "Random Walks, Martingales and the OTC" in which the authors validated the weakly efficient form of the efficient market hypothesis (1973). After stating that "the evidence overwhelmingly shows that security returns are independent over time," the authors propose an extension to the EMH in which over-the-counter (OTC) securities would be used in

6 "Rational arbitrageurs" can be classified as individuals who rationally exploit price differentials (deviation from fundamental value) in order to obtain trading advantage.

place of stocks. A more direct method is used to investigate the hypothesis in this study: serial correlation coefficients are computed for a set of 253 securities, along with the use of distribution-free runs testing, to test for autocorrelation. 12.3% of the serial correlation coefficients were significantly non-zero at the 5% level under normality, but this result is discounted as flawed for a number of reasons: variable error, covariance with the aggregate market trend, and the normality assumption introduced substantial bias into the estimate of ρ. The runs test fails to find a significantly non-zero proportion of securities with excessive deviation from normality. Therefore, Hagerman and Richmond fail to reject the null hypothesis that the OTC securities market is weakly efficient as posited by the efficient-markets theory; the EMH is not without empirical support.

In summation, we cannot claim that the efficient market hypothesis has explicitly failed. The theory has extensive empirical justification, as shown previously, and the EMH/CAPM duality has been very successful: the models remain essentially intact after more than thirty years of criticism. However, the strict assumptions of efficient-markets theory can appear implausible in the current trading environment and critics are numerous and vocal (Thaler 1993). Economic history is rife with examples of individually-irrational herd behavior, bubble formation along with the inevitable crash: Baker and Wurgler cite the Nifty Fifty and the Black Monday crash of 1987 as examples of rational traders gone awry, violations of the EMH in the short-run (2007). Recent macroeconomic events, particularly the 1990s "dot-com" bubble and the 2005 U.S. real estate bubble, have served to discredit the concept of efficient financial markets. The field of behavioral finance has emerged as a center of heterodox thought in this area, proposing alternative theories of stock price formation (Hirshleifer 2001).

The field of behavioral finance has propagated a number of alternative theories based around a set of common concepts, such as cognitive biases, but no consensus exists. Hong and Stein provide a list of reasons why stock prices would persistently deviate from fundamental value in "Disagreement and the Stock Market" (2007). Momentum, the continuation of upward or downward trend regardless of other factors, is the central tenet of technical analysis (momentum investing), an investment philosophy that argues for the intensive examination and mimicry of trend. The existence of stock price momentum again implies that a long-term profitable trading rule exists, which is inconsistent with the efficient-markets theory. In post-earnings (announcement) drift, returns are abnormally high/low following positive/negative news, respectively; the trading rule in this case is to buy stocks following a positive announcement. Mean reversion is equivalent to the so-called "overreaction hypothesis" of De Bondt, Thaler (1984); good/bad news in the short-run leads to losses/gains in the medium-run (3+ years). Almost all of the alternative hypotheses based on cognitive biases, such as the illusion of control, have not been explored to the extent required to become serious competition for the EMH. Hirshleifer's survey article, "Investor Psychology and Asset Pricing", is an example of how diverse and disparate the field has become; many scholars are trying to connect psychological concepts with investor behavior as manifested in stock market prices, but a proliferation of applicable theories in cognitive psychology has resulted in a rather wide range of applications to finance (2001). Each cognitive bias has been explored by a limited number of authors, so no one concept has reached the requisite critical mass, so to speak, to genuinely compete with the efficient market hypothesis.

Sources like De Bondt and Thaler "Does the Stock Market Overreact" (1984) find an overreaction effect in stock prices after a significant news announcement. Theoretically, human violation of Bayes' rule implies that traders tend to overestimate the effect of positive unexpected news; therefore, we should empirically see excessive stock gains immediately after favorable announcements. This "overreaction hypothesis" contradicts efficient-markets theory since stock prices temporarily yet persistently overestimate the actual value. Monthly data on NYSE common stock returns are used from Jan. 1926 to Dec. 1982; the authors cite problems with the use of daily data, such as the "bid-ask" effect and infrequent trading. Two groups of stocks are defined: a loser portfolio, stocks that suffered negative news in the recent past, and a winner portfolio, stocks under the influence of positive announcements; the portfolios are tracked for 2-5 years after the news event. Since an unanticipated announcement would tend to overvalue or undervalue stocks with positive or negative reports, respectively, the authors expect that winners will retreat and losers will gain during the subsequent correction. De Bondt and Thaler find empirically that this is the case; the loser portfolio substantially outperformed the winner portfolio in every case considered. Therefore, an investor could formulate a contrarian trading rule as follows: buy stocks on negative news and sell stocks on positive news. Such a strategy could be profitable in the long-run, according to the authors, due to this overreaction effect. The discovery of a profitable trading rule in the long-run implies the invalidity of the EMH as stock prices are not engaging in random walks.

Baker and Wurgler (2007) try to predict stock market returns using an index of investor sentiment, and this approach is relatively common in the discipline. The theoretical concept is that price changes reflect exogenous changes in investor sentiment, which can be measured by a derived sentiment index. In "Investor Sentiment in the Stock Market", the authors construct an investor sentiment index based on six factors which serve as proxies: trading volume, dividend premium, the closed-end fund discount, the number and preliminary returns of IPOs, and the equity share in new issues. Baker and Wurgler attempt to remove the macroeconomic components of the proxy variables in order to target solely sentiment as opposed to accounting for other exogenous changes. This index is moderately successful in terms of predicting future returns; stocks that are difficult for arbitrageurs tend to be more intensively affected by changes in sentiment. These results, when combined with the conclusions of others in behavioral finance, imply that sentiment indices can be accurate predictors of stock market returns.

Volatility in excess of changes in fundamental value is another cornerstone of the behavioral finance literature, and the topic has been discussed extensively (Thaler 1993; Shleifer 2000; Shiller 2000). As an example, consider "What Moves Stock Prices?" by Cutler, Poterba, and Summers (1989) as reproduced in Thaler (1993). After accounting for changes in publicly-available information, the authors conclude that as much as half of the variance in stock prices remains unexplained; this result rejects the null hypotheses of weak or semi-strong efficiency. Again, this result suggests the fallibility of efficient-markets theory in certain situations.

In *Irrational Exuberance*, Shiller colloquially discusses herd behavior, the idea that individual decision-making is influenced by the choices of others in what is termed an "information cascade" (2000). Such a cascade is characterized by incomplete information: since no agent knows the true fundamental value of an asset at a fixed point in time, the decisions of other agents in the previous period are used as a reference point. For example, the Oct. 1987 bull market was partially driven by money managers who wanted to continue their employment at investment firms; no manager wanted to miss out on the record gains that were perpetuated by the traders themselves (Scharfstein, Stein 1990). Alternatively, discerning the actual value of a stock takes a considerable amount of time and financial resources (Hirshleifer 2001); an investor might find it advantageous to bypass the research process by agreeing with the majority. This principle is an extension of Kirman's agent-based ant model, which is the basis of this paper's approach.

As defined by Kirman (1993), the "ant model" is a well-known agent-based model of ant colony behavior during the search for food.

The colony is exposed to two equally favorable non-exhaustible food sources (sites A and B) and pheromone trails from the initial scouts can be modeled as positive feedback (Ormerod 1998). We partition the colony's fixed[7] population into two mutually-exclusive groups: ants currently searching for food at site A (group A) and those foraging at site B (group B). The probability of a new ant selecting site A[8] is directly related to the number of ants in group A, and thus indirectly related to the number of ants in group B. However, the random chance that an ant would spontaneously and independently switch from one group to another is ever-present[9] (Kirman 1993).

The site visited by most initial scouts may become very popular due to the positive feedback mechanism, but sudden switching to the other group can occur if a cluster of ants randomly decides to investigate an alternative site (Kirman 1993). No long-run equilibrium exists, and rapid changes can still occur regardless of the time horizon due to the model's statistical qualities. Kirman's simple Markov chain is able to explain ant behavior so well because each ant is considered as an individual agent that chooses a food site in each period. Ormerod states that, "the idea that the system as a whole can be understood by the behavior of a single, representative agent is a complete non-starter" (1998); the conventional approach in economics, aggregation with the representative *homo economicus*, cannot apply here due to the ant recruitment method and its reliance on positive feedback.

Computational agent-based modeling is a relatively new simulation technique, at least in economics; see Bonabeau; Cioffi-Revilla; Gilbert and Bankes 2002. The increased availability of simulation tools has led to intensive application of this mathematical framework to a wide range of problems, such as individuals trying to leave a burning building through a single door (Bonabeau 2002). Each person is modeled as an agent with generalized behavioral rules regarding conduct in the group; for example, an individual attempting to escape from a fire might try to avoid or help others on the way to the door (Bonabeau 2002). The agent-based approach allows for precisely-defined unique actors: based on parameters, one agent may be more likely to attempt a reckless exit than another. Agent-based models are typically solved via simulation techniques because no closed-form solution can be found analytically. Therefore, we would expect that each trial of an agent-based model generates a unique solution that is not strictly reproducible if probabilistic components are involved in the modeling scheme. Parameter values are important in ABM because parameter inaccuracy can lead to large changes in model outcomes[10]; the parameters of interest are usually exogenously determined, however, making empirical comparison difficult (Kirman 1993).

A recent application of the interacting-agent approach can be seen in Arthur et al. "Asset pricing Under Endogenous Expectations in an Artificial Stock Market" (1997) as the authors construct a self-contained artificial stock market in which each trader is assigned his/her own unique bundle of pricing models. The poorly-performing models are dropped and new models are added, so each agent generates expectations based on the outcomes of his/her respective models. Therefore, expectations are internally generated, not exogenously imposed, and prices interact with expectations in a dynamic fashion.

7 Under the assumption that the colony experiences zero population growth in the short-run. A more sophisticated long-run model could express population growth as a function of the quantity of food gathered in each period.

8 Joining group A (visiting site A), leaving group B (ignoring site B).

9 The analogous stock market situation: group A is the set of optimists (bulls); group B is a collection of pessimists (bears); the ants are traders who engage in a search for return on initial investment given the known risk-reward environment and exogenous macroeconomic variables (theoretically).

10 Final index price is one such outcome, and we usually have a target for that value based on historical information. Therefore, set the expected value of final index price equal to the recorded final price in the data set in order to maximize the likelihood of achieving the actual quantity in a representative simulation run.

III. Random-walk Model

As shown by Samuelson and Mandelbrot, efficient-markets theory implies that the value of a frequently-traded stock should engage in a random walk about its fundamental value because stock prices fully reflect all available information about the expected value of discounted future earnings. Traders, who primarily concern themselves with the difference between actual and fundamental value, will quickly correct the price of an undervalued or overvalued stock. The availability of complete, accurate information implies that traders are able to integrate changes in the important earnings indicators into the stock price almost instantaneously. Stock market indices are simply bundles of individual stocks, so index value should also deviate from its fundamental value in a random-walk process, where the fundamental value of an index is the summation of the fundamental values of its component stocks. Theoretically, we have that expected returns are positive and constant (Sheffrin 1996); formally,

$$E[\tilde{Z}_t - \overline{Z}_t | I_{t-1}] = 0 \qquad (3.1)$$

$\tilde{Z}_t$ = actual return in time t,

$\overline{Z}_t$ = expected return in time t,

I_{t-1} = information set at time t-1.

The expected value (E[] operator) of actual return minus (constant) expected return given the previous information set is zero; actual return never deviates from expectations based on available information (Ibid.). If deviation from expected return does occur, this disparity should be extremely short-lived as expectations rapidly adjust.

The particular mechanism through which the stock price random-walk is transmitted can be specified in a number of equivalent ways, three of which are considered here. The simplest case of a random-walk model is the driftless case with finite up or down steps at each time interval; at each decision point, the series either increases by one or decreases by one with equal probability. "Drift" is conceptually defined as the nonrandom per-period change in the dependent variable; drift ought to be representative of long-run change or trend. Var[] is the variance operator.

$$a_0 = p_0 \qquad (3.2\text{-}3.5)$$

$$a_{n+1} = a_n \pm 1 = a_n + (-1)^{\text{Binomial}\left[1, \frac{1}{2}\right]} \text{Binomial}\left[1, \frac{1}{2}\right]$$

$$E[a_n] = p_0$$

$$\text{Var}[a_n] = n$$

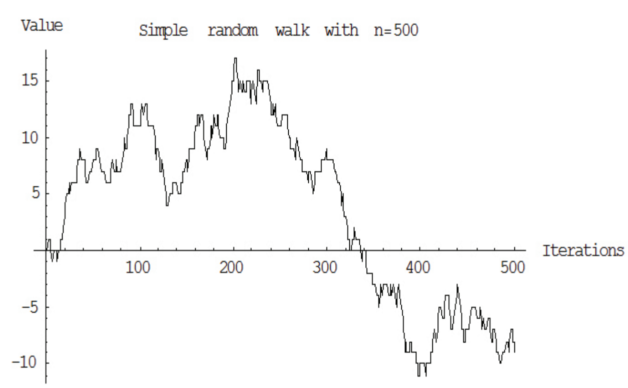
$$\{a_n - a_{n-1}\} \sim (-1)^{\text{Binomial}\left[1, \frac{1}{2}\right]} \text{Binomial}\left[1, \frac{1}{2}\right] \qquad (3.6\text{-}3.8)$$

$$E[a_n - a_{n-1}] = p_0$$

$$\text{Var}[a_n - a_{n-1}] = 1$$

Figure 3.1 shows a representative simulation of this type of stochastic process. The process is non-stationary[11] because the variance is a function of time index n. The sequence of first differences $\{a_n - a_{n-1}\}$ is stationary, however; in this case, differencing can achieve stationarity. Note that $E[\rho(a_n, a_{n-1})] = 0$ as well.[12]

Fig. 3.1: Simple random walk stochastic process.



One-dimensional random walks without drift

11 A stochastic process is said to be stationary if its probability distribution function is time-invariant; we would expect that a stationary process has time-independent moments, such as the first and second moments of mean and variance, if they exist. The concept of stationarity is important because non-stationarity implies that the underlying probabilistic process (probability density function) is changing over time.

12 Where ρ is the correlation coefficient between a_n and a_{n-1}.

are characterized by the following relations:

$a_0 = p_0$ (3.9-3.15)

$a_{n+1} = a_n + N(0,\sigma^2)$

$E[a_n] = p_0$

$Var[a_n] = n\sigma^2$

$\{a_n - a_{n-1}\} \sim N(0,\sigma^2)$

$E[a_n - a_{n-1}] = 0$

$Var[a_n - a_{n-1}] = \sigma^2$

The untransformed process is non-stationary, while the differenced sequence is stationary; $E[\rho(a_n, a_{n-1})] = 0$ since $\{a_n - a_{n-1}\} \sim N(0,\sigma^2)$.

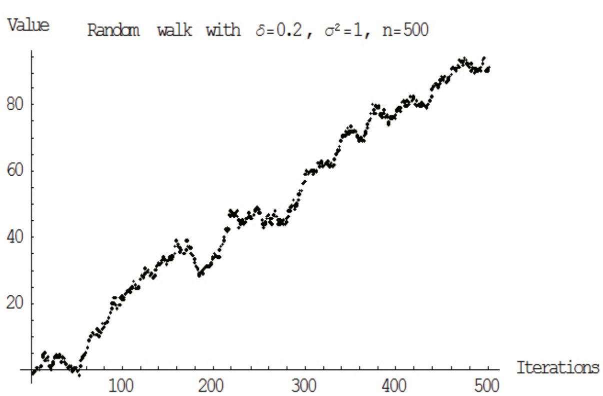
A one-dimensional random-walk process with drift δ is defined recursively as follows:

$a_0 = p_0$ (3.16-3.17)

$a_{n+1} = a_n + \delta + N(0,\sigma^2)$

$N(0,\sigma^2)$ is a normally-distributed random variable with mean 0 and constant variance σ^2. A typical result of such a process is plotted below (Figure 3.2).

Fig. 3.2: Random walk process (with drift δ).



The three random-walk processes discussed have common characteristics that can be empirically tested for regardless of the particular model specification used, and these traits are shared by all random-walk models:

(i) $E[\rho(1da_n, 1da_{n-1})] = 0$; 1d = first difference;

(ii) $\{a_n\}$ is non-stationary;

(iii) $\{\Delta a_n\}$ is stationary;

(iv) $\lim_{n\to\infty} Var(a_n) = \infty$.

Our purpose in examining multiple specifications was to draw out these useful shared traits. The presence of autoregressive behavior in the first differences of a particular time-series data set indicates that the original series was not the product of a random walk process. Additionally, testing for stationarity in the unmodified and first difference financial time-series can validate or invalidate the random walk hypothesis (Diba, Grossman 1988). Tests for non-stationarity include the augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests for unit roots; the existence of a unit root implies that the original series is non-stationary, but differencing may be used to obtain a stationary series. Therefore, tests are available that will evaluate the soundness of the EMH as manifested in the random walk hypothesis using data from the Hang Seng and Nikkei 225 stock market indices. Drift parameter δ can be interpreted as the long-run trend regarding the value of the index as determined by corporate finance and macroeconomic fundamentals; the EMH posits that stock prices will engage in a random walk around this trend as all available information has already been integrated into the price.

IV. Agent-based Model

As summarized by Ormerod in *Butterfly Economics*, the agent-based approach to time-series modeling defines a finite number of groups that probabilistically interact with each other according to simple behavioral rules (1998). ABM treats each individual separately, and although the behavioral rules may be uniform across individuals and groups, large-scale simplification and aggregation is impossible. Many economic models can be solved by resorting to the representative agent approach and aggregating across a particular group, but agent-based models are defined by the inter-agent or inter-group dynamic; using a single agent to model the behavior of a cluster of agents will remove the micro-level mechanics that enable person-to-person interaction. The statistical nature of ABM implies that each model trial will generate a unique outcome because random variables are embedded into the recursive equations; however, the outcome of the n^{th} period takes the previous *n-1* outcomes as given. Therefore, the quasi-deterministic ABM approach requires the use of simulation and variation of parameters in order to reach any well-supported conclusions as one cannot test directly for agent-based behavior in financial time-series. The ant model framework (two groups, four flows between them) was adapted from Kirman's "Ants, Rationality, and Recruitment" (1993). We will now proceed to the development of the agent-based "ant trader" investor sentiment model.

n : time index

a_n : buyer/bullish/optimistic group of traders

b_n : seller/bearish/pessimistic group of traders

$c_n = a_n - b_n$: difference in group sizes (net optimism, sentiment index)

P(n) : stock (index) price as a function of time

N : total number of traders participating in the market

$$a_{n=0} = a_o \qquad (4.1\text{-}4.4)$$

$$b_{n=0} = b_o$$

$$N = a_o + b_o = a_n + b_n$$

$$\forall n \in [0,\infty) \ \exists a_n, b_n \ \text{s.t. } N = a_n + b_n$$

p_1 : random a→b switch probability, $p1 \in (0,1)$

p_2 : a→b persuasion parameter, $[p_2(b_n)] \in (0,1)$

p_3 : random b→a switch probability, $p3 \in (0,1)$

p_4 : b→a persuasion parameter, $[p_4(a_n)] \in (0,1)$

p_5 : translational parameter (c(n) to ΔP(n)), $p5 \in (-\infty,\infty)$

$$p(a \rightarrow b)_n = p_1 + p_2[b(n-1)] \qquad (4.5\text{-}4.10)$$

$$p(b \rightarrow a)_n = p_3 + p_4[a(n-1)]$$

$$a_n = \text{Binomial}[a_{n-1}, 1- p(a \rightarrow b)_n] + \text{Binomial}[b_{n-1}, p(b \rightarrow a)_n]$$

$$b_n = \text{Binomial}[a_{n-1}, p(a \rightarrow b)_n] + \text{Binomial}[b_{n-1}, 1- p(b \rightarrow a)_n] = N - a_n$$

$$P(0) = p_o$$

$$P(n) = P(n-1) + p_5[c(n)]$$

Balanced parameters ($p_1 = p_3$, $p_2 = p_4$) will result in zero variance in the 0^{th} period and constant variance in all subsequent periods. Imbalanced parameters generate a monotonically increasing/decreasing sequence {Var[P(n)]} as $n \rightarrow \infty$; therefore, parameter balance is required for variance stationarity as well. The following conclusions regarding the "ant trader" model are consequences of our expressions for the expected value and variance of the P(n) terms.

(i) $\Delta P(n) = P(n) - P(n-1) = p_5 c_n = p_5(2a_n - N)$ from the definition of the agent-based model.

(ii) $E[\Delta P(n)] = 0$ with balanced parameters;

(iii) $E[\Delta P(n)]$ is non-constant with imbalanced parameters.

(iv) $Var[\Delta P(n)] = 4(p_5)^2 Var[a_n] = c$ with balanced parameters (c = constant).

(v) $Var[\Delta P(n)]$ is non-constant with imbalanced parameters.

(vi) Therefore, the {Δ P(n)} series is non-stationary with imbalanced parameters.

Our simulation results indicate that the $\{a_n\}$ series is (approximately) normally distributed with mean $\mu=(N/2)$, as is $\{\Delta a_n\}$ with mean $\mu=0$, given that the parameters are balanced. As a result, $\{\Delta p_n\}$ is also normally distributed; again, this only applies for balanced parameters, which cannot be used with non-zero trend.

Our "ant trader" model has the following theoretical properties:

Fig. 4.1: Stationarity of agent-based model (based on parameters).

Series	Balanced	Imbalanced
{P(n)}	stationary	non-stationary
{ΔP(n)}	stationary	non-stationary

Since both indices have sample means that are not equal to their initial values, imbalanced parameters are required in order to match this long-run upward trend. Consequently, this implies that the stationarity of first-differences is the decisive difference between the EMH-based random walk model and the ant trader model; stationarity of the non-differenced series will not be decisive. If $E[\rho(P(n), P(n-1))] \neq 0$, then only the ant trader model can adapt via parameter fitting and the random-walk model is inconsistent with the data. The need to raise the asset (index) price if buyers outweigh sellers in period *t* is an expression of simple supply and demand equilibration; the price adjusts according to investor sentiment in order to clear the market in every period.

V. Data

Hang Seng and Nikkei 225 stock index data were obtained over the periods 1987-2007 (5080 obs.) and 1984-2007 (5797 obs.); summary statistics, bivariate correlations, and graphical analysis are provided here. The two indices in question were chosen for their instability during the period, coupled with the fact that a minority of authors has used data from Asian markets before and after the 1997 financial crisis; i.e. there is sufficient variation for the models to explain. The following figures (Tables 5.1-5.5; Figures 5.1-5.2) characterize the data sets in terms of variable definitions, summary statistics, bivariate correlations, and the density of returns.

Table 5.1: Variable definitions.

Variable	Units	Definition
Returndaily	unitless	Daily percentage change in index price;
Open	HKD/Yen	Index price at open of trading session;
High	HKD/Yen	Highest index price during trading day;
Low	HKD/Yen	Lowest index price during trading day;
Close	HKD/Yen	Index price at close of trading session;
Change	HKD/Yen	Daily change in index price, unweighted;
Volume	shares	Number of shared traded during trading day.

Note that Returndaily is defined as the percentage change in index price; only active trading days were recorded in the data set, so weekends and holidays are excluded. Therefore, bias is introduced since we would expect above-average volatility following weekends as new information needs to be integrated into stock prices.

Table 5.2: Summary statistics for the Hang Seng index variables.

Variable	Obs.	Mean	Std. Dev.	Variance	Skewness	Kurtosis
Returndaily	5079	0.0549875	1.667655	2.781074	-1.929032	48.46569
Open	4516	10559.73	4562.354	$2.08*10^7$	0.0279019	2.569677
High	4516	10636.03	4588.401	$2.11*10^7$	0.0207726	2.566115
Low	4516	10474.56	4527.343	$2.05*10^7$	0.0331988	2.572902
Close	5080	9700.088	4940.557	$2.44*10^7$	0.1162412	2.233254
Change	5080	0.0552217	1.667572	2.780795	-1.929172	48.46553
Volume	1496	451774.7	460654.9	$2.12*10^{11}$	3.358333	17.3229

Hang Seng daily return is skewed to the right (negative skewness), thus the distribution is highly peaked and asymmetric.

Table 5.3: Summary statistics for the Nikkei 225 index variables.

Variable	Obs.	Mean	Std. Dev.	Variance	Skewness	Kurtosis
Returndaily	5796	0.0166975	1.364516	1.861903	0.0557901	10.14701
Open	5146	18592.2	6449.565	$4.16*10^7$	0.7960269	3.457427
High	5146	18723.54	6472.333	$4.19*10^7$	0.7852155	3.436237
Low	5146	18450.42	6422.845	$4.13*10^7$	0.805568	3.473681
Close	5797	17921.21	6408.639	$4.11*10^7$	0.9091101	3.604324
Change	5797	0.0166881	1.364399	1.861584	0.0558421	10.14902
Volume	1275	75584.78	187426.1	$3.51*10^{10}$	2.328616	7.45228

Daily return of the Nikkei has no skew, is relatively symmetric about zero, and is peaked.

Table 5.4: Hang Seng bivariate correlation coefficients.[13]

Hang Seng	Close	Close1d	Close2d	Close3d	Returndaily	Return1d	Return2d	Return3d	Volume	Time
Close	1									
Close1d	0.0234	1								
Close2d	-0.0016	0.6918	1							
Close3d	0.0007	-0.3709	-0.854	1						
Returndaily	0.0072	0.883	0.6148	-0.333	1					
Returndaily1d	-0.001	0.6069	0.8833	-0.7573	0.6943	1				
Returndaily2d	-0.0008	0.3291	0.7542	-0.8841	0.3766	0.8557	1			
Returndaily3d	-0.0012	0.1442	0.5227	-0.7964	0.1649	0.5934	0.9014	1		
Volume	0.7354	-0.0094	-0.025	0.0049	-0.0013	-0.0192	-0.0059	-0.0009	1	
Time	0.8819	0.0074	-0.0004	-0.0003	-0.0073	-0.0001	0.0001	0.0001	0.5814	1

13 Close1d = first difference of closing price, Returndaily1d = first difference of daily return, etc.

Table 5.5: Nikkei 225 bivariate correlation coefficients.

Nikkei 225	Close	Close1d	Close2d	Close3d	Returndaily	Return1d	Return2d	Return3d	Volume	Time
Close	1									
Close1d	0.0164	1								
Close2d	0.0003	0.7081	1							
Close3d	-0.001	0.3865	0.8566	1						
Returndaily	0.0133	0.953	0.6821	0.3783	1					
Returndaily1d	0.0005	0.6752	0.955	0.8226	0.7131	1				
Returndaily2d	-0.0007	0.3717	0.8177	0.9556	0.3973	0.86	1			
Returndaily3d	-0.0011	0.1753	0.575	0.863	0.1914	0.6088	0.9059	1		
Volume	-0.4401	-0.0506	0.0067	0.003	-0.0597	0.0083	0.0039	0.004	1	
Time	-0.4416	-0.0127	-0.0007	-0.0003	-0.0131	-0.0007	-0.0003	-0.0002	-0.592	1

The correlation coefficients between Returndaily and its differences (Returndaily1d, Returndaily2d, and so on) are negatively related to difference number for both indices, as expected, in Tables 5.4 and 5.5.Upon examination of Figures 5.1 and 5.2, we cannot claim that daily returns are normally distribued; this contradicts the efficient markets hypothesis by default as normality is assumed.

Figure 5.1: Distribution of Hang Seng daily return.[14]

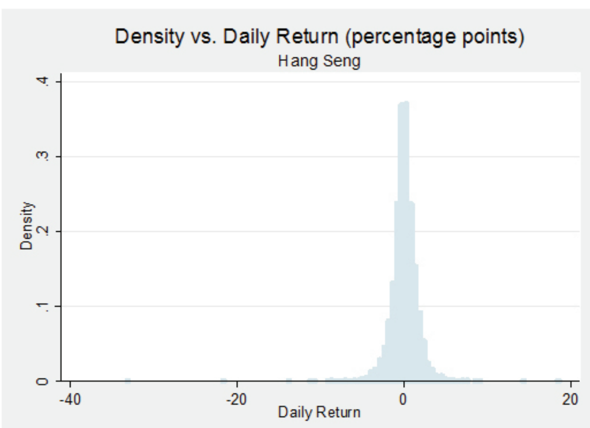
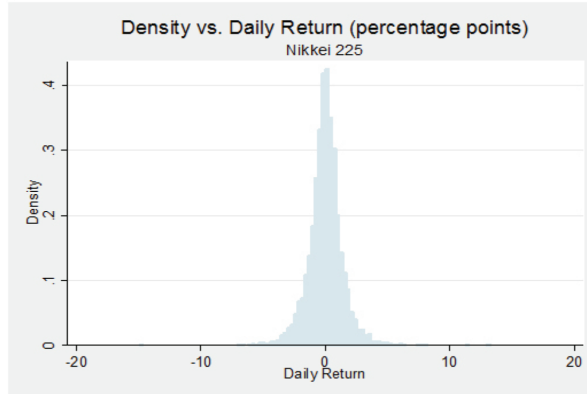


Figure 5.2: Distribution of Nikkei 225 daily return.[15]



VI. Statistical Testing Procedure

The efficient market hypothesis will be examined through two batteries of econometric testing: stationarity/unit roots[16] (ADF, PP tests) and autocorrelation (degree of autoregressive behavior). A random walk model has certain statistical properties that can be tested for empirically: non-stationarity of the untransformed series, stationarity of differences, and independence of successive values. The drift term δ will be used to approximate the linear trend in fundamental value without actually attempting a regression that tries to derive fundamental value for a number of reasons: previous attempts at divining "true" fundamental value have not been entirely fruitful due to a number of innate causal reasons, such as stock prices themselves causing changes in fundamental value and the use of stock prices as a macroeconomic indicator. Drift δ will be selected such that final price matches the expectation value of the last term of the simulated series. Work done in this area will be limited and representative, and the framework used was discussed in the theory section

Simulation data derived from the ant trader model will undergo stationarity tests and the examination of autoregressive traits. We expect to find that the untransformed series and

14 Hang Seng returns are not normally distributed; the Shapiro-Wilk normality test indicates that, with the null hypothesis of non-normality, P>Z=0.000. The Shapiro-Francia test agrees with this result.

15 Nikkei 225 returns are also not normally distributed when using the same normality tests.

16 ADF: Augmented Dickey-Fuller test, PP: Phillips-Perron test; both are stationarity tests.

first differences in price are non-stationary, and significant autoregressive behavior can only be explained by the agent-based model. Empirical modeling in this case is the application of previously discussed theoretical models to the specific case of Nikkei 225 (1984-2007) and Hang Seng (1987-2007) index prices using parameter estimation by fitting expected mean and variance to the data. Statistical routines will be executed in accordance with *Stata* 10 definitions.

VII. Results

We first need to obtain a theoretical estimate for the random walk model's mean and variance. Using this estimate, E[μ] from the model can be matched to the sample mean in real-world cases (Hang Seng, Nikkei 225). Regarding the agent-based model, only a large-sample simulation could provide the necessary relationship between parameter values and μ or σ2, so such a simulation is attempted. Given the poor results of this effort, *ceteris paribus* studies are done in which a selected parameter is allowed to vary within ±5% of its initial value. Next, E[μ] and E[σ2] are derived for the agent-based model using the definitions established in Section IV; again, the purpose of this is to match E[μ] to the sample mean by varying the model's parameters: p_1, p_2, …, p_5. Finally, stationarity testing is done using *Stata* 10 routines in order to determine which model best reflects market conditions for the two indices.

The random walk with drift and "ant trader" models are fit to the data using the first and second central moments of mean and variance, respectively. Therefore, we need to obtain E[μ] and E[σ²] in each case in order to plausibly match real-world behavior. For the random walk model, these expected values can be found analytically; *k* stands for the number of model iterations.

$$E[\mu] = E[\tfrac{1}{k}\textstyle\sum_{n=0}^{k} a_n]$$

$$= \tfrac{1}{k}\textstyle\sum_{n=0}^{k} E[a_n]$$

$$= \tfrac{1}{k}\textstyle\sum_{n=0}^{k} (p_0 + n\delta)$$

$$E[\mu] = p_0 + \frac{(k+1)\delta}{2}$$

$$E[\sigma^2] = E[\tfrac{1}{k-1}\textstyle\sum_{n=0}^{k} (a_n - \mu)^2]$$

$$= \tfrac{1}{k-1}\textstyle\sum_{n=0}^{k} (E[a_n] - E[\mu])^2$$

$$= \frac{\delta^2}{k-1}\textstyle\sum_{n=0}^{k} \left(n - \frac{n+1}{2}\right)^2$$

$$= \ldots$$

$$E[\sigma^2] = \frac{\delta^2(2k^2 - 5k + 6)(k+1)}{96(k-1)}$$

Since *k* and p_0 are taken as given based on the data, we can only change the drift term δ in order to retain consistency with the sample mean and variance. The assumption made here was that the random (non-drift) component is normally distributed with mean 0 and variance 1; additional flexibility requires changing these parameters, which will not be considered here. δ will be selected to match the sample mean, so this current scheme allows for no control over variance.

Simulations were run using the agent-based model with randomly drawn parameter values uniformly distributed on finite intervals. p0 and k were fixed based on the index used (Hang Seng or Nikkei); N was fixed at 100; p_1, p_2, p_3, p_4, and p_5 were allowed to vary as Figure 7.1 shows.

Fig. 7.1: Allowed parameter ranges, m=2450 simulation.

$p_1, p_3 \in [0.01, 0.1]$
$p_2, p_4 \in [0.0005, 0.005]$
$p_5 \in [1, 10]$

The simulation program, executed in *Mathematica* 6.0, calculated the first four central moments (mean, variance, skewness, and kurtosis) for each trial. Each trial was represented by a line in the data file, which recorded the central moments coupled with relevant parameter values, as shown in Figure 7.2.[17]

Fig. 7.2: Mathematica-generated data file.

ABMdata.txt:

$\{p_{11}, p_{21}, p_{31}, p_{41}, p_{51}, \mu_1, \sigma^2{}_1, \gamma_{11}, \gamma_{21}\}$

$\{p_{12}, p_{22}, p_{32}, p_{42}, p_{52}, \mu_2, \sigma^2{}_2, \gamma_{12}, \gamma_{22}\}$

...

$\{p_{1m}, p_{2m}, p_{3m}, p_{4m}, p_{5m}, \mu_m, \sigma^2{}_m, \gamma_{1m}, \gamma_{2m}\}$

We then attempted to regress these moments on the recorded parameter values for each draw using Stata 10. The following results are for the Nikkei 225 simulation with $p_0 = 9927$, $k = 5796$, $N = 100$, $p_5 = 5$ fixed for all trials; $m = 2450$ total trials were run, so the data file had 2450 lines. Sample mean was regressed on parameter values p_1, p_2, p_3, and p_4 initially; the result, as seen in Table 7.1, was surprising as no variables are significant at the 0.10 level, although p_4 is close. However, even this weakly-significant variable is contradictory because all parameters should matter theoretically, according to our definitions.

Table 7.1: Regression results, mean = f(p1,p2,p3,p4).

Source	SS	df	MS		
Model	2.384*10^12	4	5.961*10^11	N	2450
Residual	1.483*10^15	2445	6.068*10^11	P > F	0.415
Total	1.486*10^15	2449	6.067*10^11	Adj. R^2	-0.000
				RMSE	7.8*10^5

Mean	Coef.	Std. Err.	t	P>|t|	95% CI LB	95% CI UB
p1	-52871.02	605941.1	-0.09	0.93	-1241082	1135340
p2	-2035137	1.21*10^7	-0.17	0.867	-2.58*10^7	2.17*10^7
p3	717052.9	611050.5	1.17	0.241	-48177.3	1915283
p4	1.98*10^7	1.22*10^7	1.62	0.106	-4187150	4.38*10^7
constant	-78461.9	69319.92	-1.13	0.258	-214393.7	57469.94

Regressing variance on the set of parameter values $\{p_1, p_2, p_3, p_4\}$ generated a similar result, as seen in Table 7.2. The constant term was the most significant, indicating high variance regardless of parameters. We did achieve significance of p_2 at the 0.10 level; again, the other parameters are not significant. This implies that parameters reduction in the agent-based model may be possible, as only a subset of the available parameters is important in explaining simulation mean and variance. Please see the appendix for a full listing of attempted regressions (Tables A.1-A.4).

Table 7.2: Regression results, variance = f(p1,p2,p3,p4).

Source	SS	df	MS		
Model	9.122*10^22	4	2.280*10^22	N	2450
Residual	6.931*10^25	2445	2.834*10^22	P > F	0.5222
Total	6.940*10^25	2449	2.834*10^22	Adj. R^2	-0.0003
				RMSE	1.7*10^11

Variance	Coef.	Std. Err.	t	P>|t|	95% CI LB	95% CI UB
p1	3.37*10^10	1.31*10^11	0.26	0.797	-2.23*10^11	2.90*10^11
p2	-4.34*10^12	2.62*10^12	-1.66	0.098	-9.47*10^12	7.99*10^11
p3	-5.63*10^10	1.32*10^11	-0.43	0.670	-3.15*10^11	2.03*10^11
p4	1.06*10^12	2.64*10^12	0.40	0.687	-4.12*10^12	6.25*10^12
constant	2.13*10^11	1.50*10^10	14.22	0.000	1.84*10^11	2.42*10^11

This null result implied that our model specification was incorrect. The probabilistic definition of the "ant trader" model suggested that only the relative parameter values were important, i.e. the ratios Randomratio:= p_3 / p_1 and Persuasratio:= p_4 / p_2. Independent variables Randomdiff:= $p_3 - p_1$ and Persuasdiff:= $p_4 - p_2$ could have explanatory power if the difference in parameters was influential. Although most regressions did not assign significance to these new variables, the persuasion ratio was significant at α=0.05 when predicting sample mean. Our simulation results yielded one definite conclusion: model outcomes "explode" when the parameters become imbalanced; Highly unequal parameters ($p_1 >>> p_3$, $p_2 >>> p_4$, or vice versa) lead to large sample means of $\pm 10^6$ or more. A plot of frequency vs. mean for the simulation data (m = 2450) clearly shows a twin-peaked distribution that is skewed away from zero and towards extreme values (please see Appendix, Figure A.1). These mixed results suggest that another analytical tool is necessary, specifically variation of parameters around an arbitrary starting point, in order to maintain relative stability while simultaneously exploring individual parameter effects on the central moments.

Given a starting point of $\{p_1, p_2, p_3, p_4, p_5, N\} = \{0.05, 0.001, 0.05, 0.001, 5, 100\}$, parameters p_1, p_2, p_3, and p_4 were varied individually with tolerance ± 5% *ceteris paribus*. Initial price p_o and duration k (total number of iterations) were specified according to real-world Hang Seng

17 γ_1 = skewness, γ_2 = kurtosis.

and Nikkei 225 data. Parameter values used are specified in Table 7.3, as shown below:

Table 7.3: Allowed ranges, variation of parameters.

Parameter	Hang Seng	Nikkei 225
p_1	0.05 ± 5%	0.05 ± 5%
p_2	0.001 ± 5%	0.001 ± 5%
p_3	0.05 ± 5%	0.05 ± 5%
p_4	0.001 ± 5%	0.001 ± 5%
p_5	5	5
N	100	100
p_0	2583	9954
k	5080	5797

Samples of size n=50 were used for each set of parameters, i.e. fifty runs of the model each time, where the parameters p_1, …, p_4 were discretely varied from p_x-5% to p_x+5% in intervals of 1%. The averages of the first four central moments were recorded for each parameter set across the fifty model runs. Results were graphed with SSE-minimizing linear (sample mean case) and polynomial of order 2 (sample variance case) interpolating functions. We will consider the E[μ] vs. p_2 case as representative of the results obtained. In this case, simulation results indicated that E[μ] is indirectly related to p_2 and therefore directly related to p_4; the relationships in Figure 7.3 and Figure 7.4 are subsequently linear. Variance was found to be directly related to the difference $|p_4 - p_2|$; increased deviation of p_2 from the fixed value of p_4 results in an exponential increase in the sample variance. As expected, p_1 was also found to be indirectly related to E[μ], which implies that p_3 is directly related in an analogous fashion. The difference $|p_3-p_1|$ affects sample variance directly, behaving in the same way as $|p_4-p_2|$. Parameter p_5 was not a significant predictor of sample mean or variance within the 5% tolerance.

Fig 7.3: E[μ] vs. p_2, Hang Seng model.

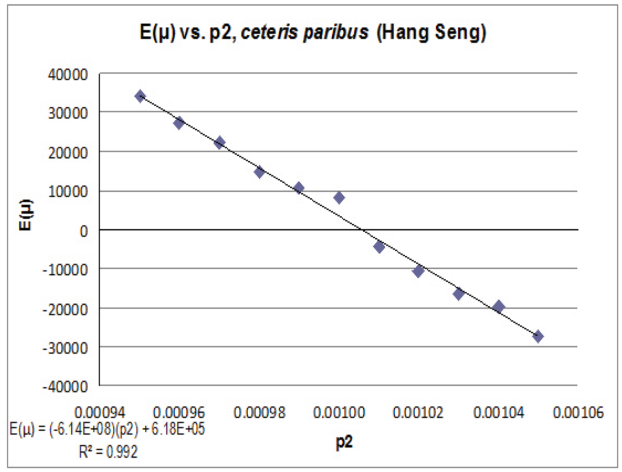
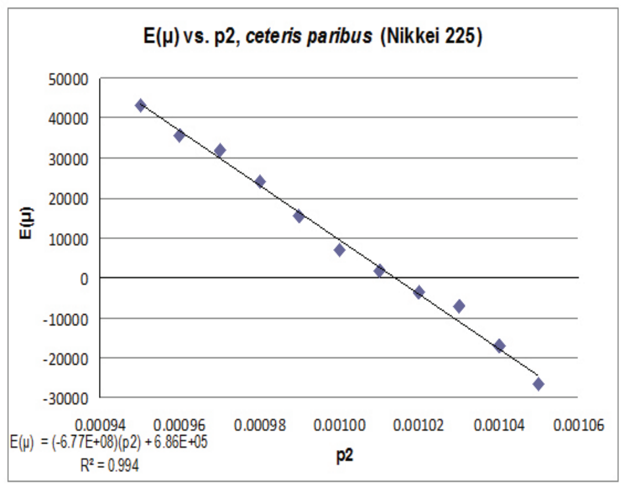


Fig 7.4: E[μ] vs. p_2, Nikkei 225 model.



We can rely on our explicit forms of $E[a_n]$ and $Var[a_n]$ in order to compute the needed quantities E[μ] and E[σ²]; the rest follows directly in Figure 7.5.

Fig. 7.5: Expectation values for first and second (central) moments, "ant trader" model.

$$E[\mu] = \frac{1}{k}\sum_{n=0}^{k} E[a_n]$$

$$E[\sigma^2] = \frac{1}{k-1}\sum_{n=0}^{k} (E[a_n] - E[\mu])^2$$

Recall that $E[a_n]$ has been recursively defined previously. Therefore, given a parameter set, we can use *Mathematica* to solve for these expectation values, which will be set equal to sample mean and variance. Now that $E[\mu(p_1,p_2,...)]$ and $E[\sigma^2(p_1,p_2,...)]$ are well-defined for both the random walk and agent-based models, we can proceed to parameter selection, point-by-point simulation, stationary testing, and comparison to the actual data. Table 7.4 lists the parameters selected for each model, based on the index, in order to match the first two central moments as accurately as possible; sample mean was prioritized over variance. Thirty trials will be computed for each set of parameter values, and stationarity tests will be individually applied to each run. Augmented Dickey-Fuller (ADF) and Phillips-Peron (PP) unit root tests are used. All routines were performed in *Stata* 10, with the results reported in Tables 7.5 – 7.6. The ADF test has no lags or drift/trend term, and the PP test uses the default number of lags (nearest integer value of $4(k/100)^{2/9}$, where the series is k periods long (k iterations in the model).

Table 7.4: Parameters for stationarity testing.

Parameter	Hang Seng (RWM)	Nikkei 225 (RWM)	Hang Seng (ABM)	Nikkei 225 (ABM)
δ	2.8014	2.7568		
p1			0.04944	0.04945
p2			0.001	0.001
p3			0.05	0.05
p4			0.001	0.001
p5			5	5
N			100	100
p0	2583	9954	2583	9954
k	5080	5797	5080	5797

Table 7.5: Stationarity tests, Hang Seng Index, $\{p_n\}$ series.[18]

Run	ABM: ADF	ABM: PP	RWM: ADF	RWM: PP
1	0.900	0.767	0.932	0.932
2	0.997	0.890	0.708	0.709
3	0.000	0.134	0.950	0.950
4	0.800	0.724	0.510	0.517
5	0.980	0.804	0.966	0.967
6	0.995	0.892	0.924	0.925
7	0.847	0.787	0.292	0.298
8	0.992	0.858	0.974	0.974
9	0.627	0.753	0.982	0.982
10	0.522	0.653	0.994	0.995
11	1.000	0.993	0.552	0.573
12	0.163	0.571	0.881	0.881
13	1.000	0.994	0.993	0.992
14	0.906	0.789	0.990	0.991
15	0.997	0.953	0.999	0.999
16	0.965	0.861	0.944	0.945
17	0.995	0.662	0.581	0.605
18	0.726	0.750	0.962	0.962
19	1.000	0.998	0.956	0.956
20	0.061	0.246	0.977	0.977
21	1.000	0.986	0.991	0.992
22	0.413	0.510	0.969	0.969
23	0.829	0.773	0.267	0.253
24	1.000	1.000	0.798	0.787
25	0.980	0.654	0.850	0.846
26	0.986	0.863	0.997	0.997
27	0.971	0.930	0.995	0.995
28	0.869	0.611	0.986	0.986
29	0.858	0.450	0.951	0.951
30	0.711	0.572	0.986	0.986

Table 7.6: Stationarity tests, Nikkei 225 Index, $\{p_n\}$ series.

Run	ABM: ADF	ABM: PP	RWM: ADF	RWM: PP
1	0.995	0.940	0.979	0.979
2	0.901	0.639	0.751	0.759
3	0.000	0.027	0.971	0.971
4	0.090	0.402	0.632	0.623
5	0.964	0.788	0.996	0.996
6	0.789	0.694	1.000	1.000
7	0.998	0.972	0.993	0.993
8	0.991	0.903	0.988	0.988
9	0.997	0.893	0.995	0.995
10	0.187	0.394	0.706	0.716
11	0.996	0.928	0.994	0.993
12	0.997	0.890	0.973	0.972
13	0.994	0.880	0.912	0.913
14	0.076	0.576	0.776	0.784
15	0.988	0.952	0.877	0.876
16	0.050	0.519	0.960	0.960
17	0.100	0.367	0.992	0.992
18	1.000	0.998	0.988	0.987
19	0.000	0.025	0.891	0.889
20	0.926	0.812	0.977	0.977
21	0.963	0.883	0.984	0.984
22	0.688	0.578	0.976	0.977
23	0.001	0.173	0.923	0.924
24	0.788	0.750	0.948	0.948
25	1.000	0.997	0.944	0.944
26	0.375	0.481	0.521	0.526
27	1.000	0.994	0.951	0.951
28	0.998	0.955	0.584	0.627
29	0.000	0.257	0.863	0.862
30	0.005	0.154	0.893	0.892

The ADF and PP test results for the differenced series all had p-values of 0.000 or less; therefore, the first differences are stationary without exception in all cases. One possible explanation for this result regarding the agent-based model, as this was expected for the random walk model, is that our parameters were too close to the balanced case to make much of an impact. Test results imply that the ant trader model was biased towards stationarity even in the price series, which is a surprising result that goes against our expectations and the empirical realities of the data. Table 7.7 attempts to summarize the results of the stationarity tests; the major implication is that the random walk model was more akin to the actual data in many respects. Since the actual indices were non-stationary in the $\{p_n\}$ series and stationary in the $\{\Delta p_n\}$ series, the agent-based model presents a problem when its $\{p_n\}$ terms are stationary. Although the agent-based model generated a more realistic estimate for test p-values in the N225 case, the random walk model is 100% accurate in predicting stationarity. These mixed results suggest the use of one last metric.

Table 7.7: Stationarity test results, summary, model comparison.

n=30 (per model)		ADF: {p} series	PP: {p} series	ADF: {Δp} series	PP: {Δp} series
Hang Seng		0.863	0.851	0.000	0.000
Nikkei 225		0.425	0.451	0.000	0.000
HSI: ABM average		0.803	0.748	0.000	0.000
HSI: RWM average		0.862	0.863	0.000	0.000
N225: ABM average		0.629	0.661	0.000	0.000
N225: RWM average		0.898	0.900	0.000	0.000
% stationary, ABM HSI	α = 0.10	6.67%	0%	100%	100%
	α = 0.05	3.33%	0%	100%	100%
	α = 0.01	3.33%	0%	100%	100%
% stationary, RWM HSI	α = 0.10	0%	0%	100%	100%
	α = 0.05	0%	0%	100%	100%
	α = 0.01	0%	0%	100%	100%
% stationary, ABM N225	α = 0.10	30.00%	6.67%	100%	100%
	α = 0.05	20.00%	6.67%	100%	100%
	α = 0.01	16.67%	6.67%	100%	100%
% stationary, RWM N225	α = 0.10	0%	0%	100%	100%
	α = 0.05	0%	0%	100%	100%
	α = 0.01	0%	0%	100%	100%

Autocorrelation plots of returns can provide qualitative information regarding the adherence of a model to the empirical ideal. Again, the random walk model outperformed the ant trader model in this dimension of comparability; please see the appendix, Figures A.2-A.5. The RWM generates alternating, seemingly random correlation coefficients between the nth period and previous periods, which is in accordance with empirical reality. However, the ant trader model exhibits strong autocorrelation in returns which is not present in daily data. We are forced to conclude that the random walk model has outperformed the agent-based model based on the metrics chosen and the subsequent results.

VIII. Conclusion

When compared to the ant trader model, the random-walk model is more consistent with the data available on the Hang Seng and Nikkei 225 stock market indices over the 1987-2007 and 1984-2007 periods, respectively. This result cannot be generalized to other markets as various empirical papers have found inconsistent results regarding the efficient-markets theory; some financial markets appear to be efficient in the short/long run, while others are not (Worthington, Higgs 2003). Therefore, our results only apply to the particular situation examined and are possibly strictly a byproduct of the data at hand, which includes the 1997 Asian financial crisis. We did not want to selectively isolate any financial crises in order to make the stationarity tests as realistic as possible, and an obvious extension of this

18 Null hypothesis: existence of a unit root (non-stationary), p-values are reported, 5% critical value: -2.86.

work is the consideration of different periods of time. It is plausible that the market operates efficiently over certain time scales and not others; an adjustment period immediately following a financial disaster may temporarily inhibit market efficiency, for example. The results in this case imply that efficient-markets theory cannot be challenged on empirical grounds using stationarity when the comparison group is simulation data generated using the ant trader model. Although the EMH assumptions are unpalatable, inflexible, and unrealistic, the resultant simulation data are consistent with actual data when using return autocorrelations and stationarity as cross-model comparative tools. A number of useful properties of the agent-based model have been established, and variation of parameters yielded insight into the underlying agent-agent dynamic. We hope to continue to improve on the agent-based approach[19] as manifested in the ant trader model as this concept is still in its infancy when compared to the thirty years of refinement that the random walk model has undergone.

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19 The next step is to allow agent-based model parameters to vary over time in response to generalized exogenous changes in consumer confidence, macroeconomic health, or consumer confidence. Autoregressive properties need to be examined more intensively with runs and variance ratio tests.

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Table A.1: Regression results, mean = f(Randomdiff, Persuasdiff), simulated data (m=2450).

Source	SS	df	MS
Model	1.4067e+12	2	7.0333e+11
Residual	1.4846e+15	2447	6.0672e+11
Total	1.4860e+15	2449	6.0679e+11

Number of obs	=	2450
F(2, 2447)	=	1.16
Prob > F	=	0.3139
R-squared	=	0.0009
Adj R-squared	=	0.0001
Root MSE	=	7.8e+05

mean	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
randomdiff	386892.3	430497	0.90	0.369	-457283.9	1231069
persuasdiff	1.07e+07	8560967	1.25	0.210	-6050560	2.75e+07
_cons	6459.175	15750.16	0.41	0.682	-24425.85	37344.2

Table A.2: Regression results, variance = f(Randomdiff, Persuasdiff), simulated data (m=2450).

Source	SS	df	MS
Model	6.9231e+22	2	3.4616e+22
Residual	6.9335e+25	2447	2.8335e+22
Total	6.9404e+25	2449	2.8340e+22

Number of obs	=	2450
F(2, 2447)	=	1.22
Prob > F	=	0.2949
R-squared	=	0.0010
Adj R-squared	=	0.0002
Root MSE	=	1.7e+11

variance	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
randomdiff	-4.58e+10	9.30e+10	-0.49	0.623	-2.28e+11	1.37e+11
persuasdiff	2.72e+12	1.85e+12	1.47	0.142	-9.10e+11	6.35e+12
_cons	2.03e+11	3.40e+09	59.61	0.000	1.96e+11	2.10e+11

Table A.3: Regression results, mean = f(Randomratio, Persuasratio), simulated data (m=2450).

Source	SS	df	MS
Model	3.3135e+12	2	1.6567e+12
Residual	1.4827e+15	2447	6.0594e+11
Total	1.4860e+15	2449	6.0679e+11

Number of obs	=	2450
F(2, 2447)	=	2.73
Prob > F	=	0.0651
R-squared	=	0.0022
Adj R-squared	=	0.0014
Root MSE	=	7.8e+05

mean	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
randomratio	4794.755	11938.55	0.40	0.688	-18615.95	28205.46
persuasratio	26921.57	11643	2.31	0.021	4090.413	49752.74
_cons	-38893.41	28545.08	-1.36	0.173	-94868.42	17081.6

Table A.4: Regression results, variance = f(Randomratio, Persuasratio), simulated data (m=2450).

Source	SS	df	MS
Model	7.4929e+22	2	3.7465e+22
Residual	6.9329e+25	2447	2.8332e+22
Total	6.9404e+25	2449	2.8340e+22

Number of obs	=	2450
F(2, 2447)	=	1.32
Prob > F	=	0.2667
R-squared	=	0.0011
Adj R-squared	=	0.0003
Root MSE	=	1.7e+11

variance	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
randomratio	-1.98e+09	2.58e+09	-0.77	0.444	-7.04e+09	3.08e+09
persuasratio	3.57e+09	2.52e+09	1.42	0.157	-1.37e+09	8.50e+09
_cons	2.01e+11	6.17e+09	32.51	0.000	1.89e+11	2.13e+11

Figure A.1: Frequency vs. sample mean for simulated Nikkei 225 data, uniformly distributed parameters $\{p_1, p_2, p_3, p_4\}$ in ranges specified in the text, m=2500. Note the local minimum near $\mu=0$.

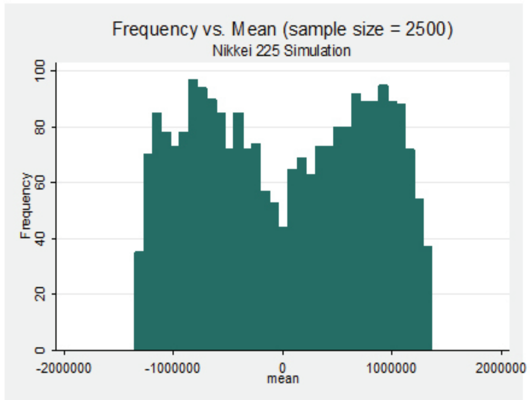

Figure A.2: Autocorrelations, 100 lags, daily return of HSI.

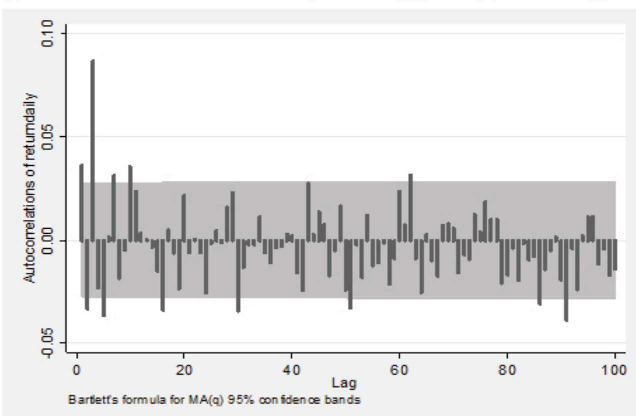

Figure A.3: Autocorrelations, 100 lags, daily return of N225.

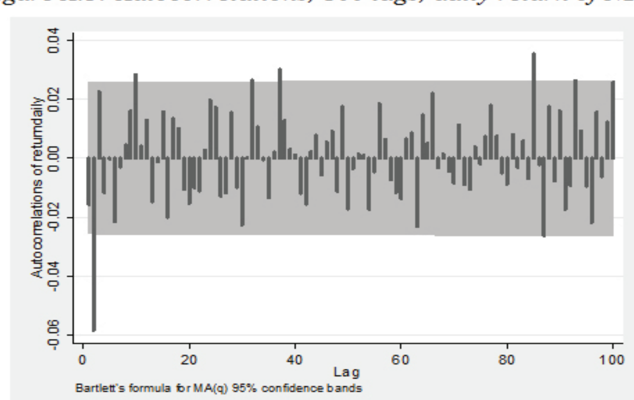

Figure A.4: Autocorrelations, 100 lags, ABM-simulated return of HSI (trial #10).

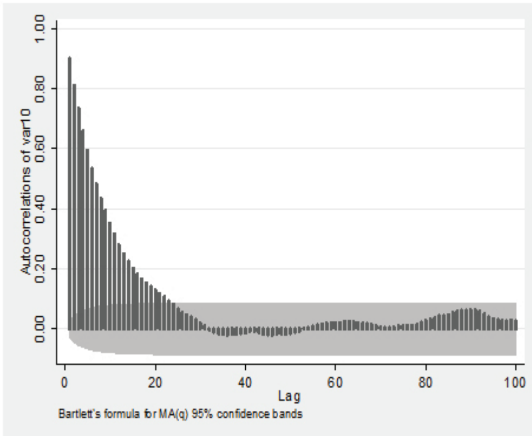

Figure A.5: Autocorrelations, 100 lags, RWM-simulated return of HSI (trial #20).

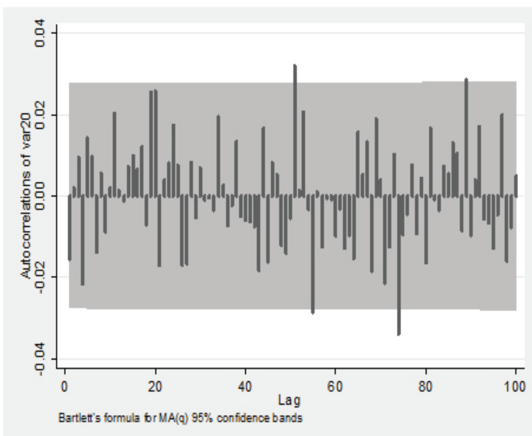

Fig. 3.1: Simple random walk stochastic process.

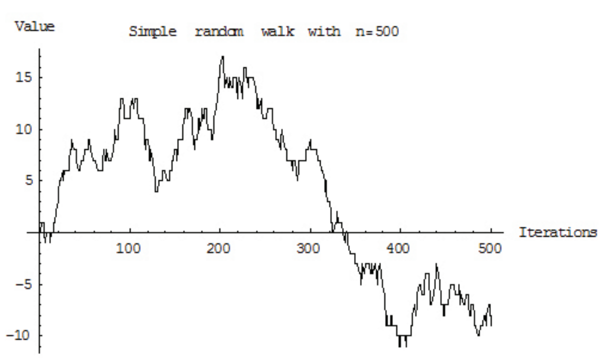

Fig. 3.2: Random walk process (with drift δ).