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Are Volatility Expectations in Different Countries Interdependent? A Data-Driven Solution to Structural VAR Identification for Implied Equity Volatility Indices

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Are Volatility Expectations in Different Countries Interdependent? A Data-Driven Solution to Structural VAR Identification for Implied Equity Volatility Indices

Abstract

Over the past couple of decades, the number of volatility indices has increased rapidly. Although the dynamics of realized volatility spillover have been studied extensively, very few studies exist that examine the spillover between these implied volatility indices. By using DAG-based structural vector autoregression, this paper provides evidence that implied volatility spillover differs from realized volatility spillover. Through solving the well-known VAR identification problem for these indices, this paper finds that Asia, more specifically Hong Kong, plays a central role in implied volatility spillover during and after the 2008 financial crisis.

Keywords

Volatility, Implied volatility, VIX, Structural VAR, Spillover, Asia

1. Introduction

Since its creation in 1993, the CBOE Volatility Index (VIX) has become widely considered as one of the best measures of investor sentiment in the world. Although the calculation of the VIX is quite complex, it has become an invaluable source of information because it is a good gauge of fear among investors. When investors open a newspaper to the stock market section or open stock market apps on their phones, chances are that they will see the current level of the VIX reported alongside other major equity indices, such as the S&P 500, the Dow Jones Industrials, or the NASDAQ. Following the success of the VIX, volatility indices based on equity indices in different countries have been created. Moreover, there has been an explosion of exchange-traded products that track the VIX, making understanding the dynamics of VIX movements more important to investors.

Despite the prevalence of research on volatility spillover (Hamao, Masulis, and Ng (1990); Engle (1994); Kanas (1998)), very little work has been done to study the dynamics of the spillover between volatility indices. The large body of previous literature on volatility spillover has calculated volatility from returns, using a GARCH-like variance equation or the standard deviation of returns. When volatility is calculated from returns, it is called realized volatility. The practical implications of studying realized volatility spillover are quite limited because there is no way for investors to gain exposure to realized volatility.¹ On the other hand, the VIX and other volatility indices that have been subsequently created are based on the implied volatility of the options traded on their respective underlying equity indices. The most important component in any option pricing model is investor's estimate of implied volatility. Consequently, unlike realized volatility, if an investor has an edge in understanding implied volatility movements, they can monetize this edge through trading options on the underlying equity index.

This paper seeks to investigate spillover among implied volatility indices based on the major equity indices around the world. Given the past literature on realized volatility spillover, there is strong evidence that implied volatility indices should be interdependent. Previous literature (Hamao, Masulis, and Ng (1990); Engle (1994); Kanas (1998)) has demonstrated that most of realized volatility spillover comes from the US. Given the popularity of the VIX, one would expect that this would be true for implied volatility spillover as well. This paper makes a strong case for a difference in spillover dynamics between implied and realized

¹ Aside from buying the replicating portfolio of a variance swap, which is costly if one seeks to have a constant vega exposure across all strikes.

volatility that merit further research, namely that the US might not be the largest source of implied volatility transmission.

In order to examine the spillover among implied volatility indices in different countries, this paper uses forecast error variance and historical decompositions from a directed acyclic graph (DAG)-based structural vector autoregression. This technique has been previously applied by Bessler and Yang (2003) and Yang and Zhou (2013) to equity indices and credit spreads, respectively. A DAG is a technique that is useful for identifying the contemporaneous casual structure between multiple time series, which provides a data-driven solution to the well-known “identification” problem in a vector autoregression (VAR) model. The use of DAG to solve the identification problem in the VAR is significantly more attractive than the widely used Cholesky factorization², which makes a strong assumption about the true data generating process and is extremely sensitive to variable ordering.³

The contribution of this paper is fourfold. First, this paper adds to the very small area of literature surrounding spillover among implied volatility indices by examining more volatility indices over a longer sample period than has previously been done. To my knowledge, only three papers (Aboura (2003); Narwal, Sheera, and Mittal (2013); Ding, Huang, and Pu (2014)) have examined spillover between these volatility indices. This paper is the first to make an attempt to solve the identification problem for volatility indices to estimate a structural VAR, while the past literature has simply used a reduced-form VAR. Most notably, this paper demonstrates that, contrary to past literature and economic intuition, the US is not the largest source of global implied volatility transmission.

Second, this paper contributes to the existing literature surrounding volatility transmission across different markets during the 2007 global financial crisis. Although there has been a large amount of studies on the financial crisis, (Duncan and Kabundi (2013); Dungey and Martin (2007); Karunanayake et al. (2010); Liow (2015); Longstaff (2010)), the use of a DAG-based structural VAR provides a more in depth look at the change in contemporaneous correlation structure of implied volatility indices. The results of this paper suggest that although the US plays a large role in spillover during the crisis, Asia plays a significant role at the start of the crisis and is an important factor in explaining the implied volatility

² This factorization assumes that the contemporaneous casual structure between variables in a VAR is lower triangular.

³ See discussion in Bessler and Yang (2003).

movements in other regions at short time horizons. Moreover, this paper demonstrates much of this spillover comes from Hong Kong, while shocks to Japan and Korea's volatility indices contribute little to the increase in implied volatility in Europe and the US. These results are somewhat different from the previous literature (Yang and Zhou (2017)), which found the US as the greatest driving factor of realized volatility spillover at most times during the 2007 crisis.

Third, the DAG-based structural VAR is described in detail in Section 4. This paper seeks to provide a more intuitive description of this technique, with the hope of encouraging the use of this data-driven solution to the identification problem that comes up often in time-series research, as an alternative to the commonly used Cholesky factorization. For a more technical discussion of this procedure, readers should consult Section 2 of Bessler and Yang (2003).

Lastly, this paper is the first to explicitly deal with the problem of stationarity when including volatility indices in a VAR framework. Previous literature has not paid attention to this issue, and I show that a log-transformation of these volatility indices is necessary for a VAR estimation to be valid.

This paper is organized as follows. Section 2 provides a review of the literature on realized volatility spillover, volatility transmission during the crisis, and the small body of literature surrounding implied volatility spillover to which this paper seeks to add. Section 3 describes the data used and their limitations. Section 4 provides a description of the empirical framework used in this paper, known as DAG-based structural vector autoregression. Section 5 examines spillover between volatility indices by region since the crisis. Section 6 investigates implied volatility transmission by region during the crisis, and Section 7 examines this transmission on the individual index level. Section 8 concludes.

2. Literature Review

Since Engle's (1982) seminal paper that introduced autoregressive conditional heteroscedasticity (ARCH) models to model volatility, the dynamics of volatility have been studied with a growing intensity. Specifically, Hamao, Masulis, and Ng (1990) were the first to examine the correlations in equity volatility in international markets and found that volatility tends to "spillover" from New York to London and subsequently from London to Tokyo. This area of literature surrounding how equity volatility is transmitted between markets has since been referred to as "volatility spillover." Engle (1994) went one step further and examined these

spillovers between New York and London on an hourly basis and found that most of the significant spillover occurs around opening and closing times.

As the presence of international equity volatility spillovers became documented, researchers began to investigate the dynamics of the spillovers more closely. Solnik, Boucrelle, and Le Fur (1996) and Kanas (1998) both showed that there is more evidence of volatility spillovers immediately following a crisis. Unlike past studies which had focused on the US market, Kanas (1998) solely looked at European stock exchanges. He found that most of the volatility spillovers among European stock indices volatility was two-directional, unlike the spillovers from the US, which tend to be one directional. Moreover, previous researchers had used Bollerslev's (1986) GARCH model for volatility, which assumes symmetry of the effects of good and bad shocks. By using Nelson's (1991) EGARCH model that allows for asymmetric effects, Kanas (1998) was able to show that spillovers exhibit strong asymmetry – bad news in one market has a greater effect on volatility in other markets than good news. Edwards and Susmel (2001) used a similar technique to demonstrate that volatility spillovers existed in emerging markets, as well as developed markets. They used Latin American and Asian stock indices, which had not yet been investigated, and found strong evidence of interdependence in volatility processes in these emerging markets as well.

Baele (2005) was the first to think about the economics of what drives volatility spillovers. He focused on developed markets by using thirteen European equity indices and one US equity index. For nearly all countries in the sample, volatility spillover had steadily increased from the second half of the 1980's. For example, the amount of variance of smaller European equity indices explained by US equity shocks rose from 15% to 27% over the sample period. Baele then attempted to identify what factors caused these spillovers. By using a ratio of market capitalization to GDP as a proxy for market development, he demonstrated that more developed markets tend to have greater volatility spillover and argued that this was because developed markets are more likely to share information than emerging markets. Other papers have found a similar result, but have argued that this is because less developed markets have more idiosyncratic volatility, which results in less interdependence and integration (Liow (2015); De Santis and Imrohorglu (1997); Duncan and Kabundi (2013)).

Following the 2007 global financial crisis and 2009 European debt crisis, researchers became interested in understanding how information was transmitted between markets during these crises. Duncan and Kabundi (2013), Dungey and

Martin (2007), and Karunanayake et al. (2010) found that during these periods of heightened volatility, most of the volatility spillover was one-dimensional from the US and sometimes Europe to less developed markets. Karunanayake et al. also found that larger indices, like those in the US and Europe, tended to have higher volatility persistence following shocks. Liow (2015) showed that although volatility spillovers fluctuate widely over time, they are significantly pronounced during crises across all asset classes. Longstaff (2010) argued that the reason volatility spillovers become more one-directional from developed markets to emerging markets during crises was primarily due to differences in liquidity across markets, rather than market development.

The dynamics of volatility spillovers between international markets has some practical relevance to portfolio managers, who have increasingly relied on international diversification as a portfolio hedge. All the aforementioned research focuses on the dynamics of realized equity volatility spillovers, where volatility is calculated from returns either directly or specified in a GARCH variance equation. In practice, there is no exchange traded product that gives exposure to realized equity volatility⁴. On the other hand, by using options and various other derivatives like volatility index futures, investors easily gain exposure to future movements of implied equity volatility. Very little research has been done to understand the dynamics implied equity volatility spillover, which I suspect is attributable to two factors. First, many implied volatility indices were created relatively recently, so it has not been possible until recent years to examine this spillover due to a lack of data. Second, realized volatility spillover among equity indices, where volatility is defined in a GARCH-like variance equation, has been studied extensively, and the drivers of this transmission are relatively well understood. Therefore, it is possible that researchers have not seen examining spillover between volatility indices as a fruitful area of research, since the drivers of implied volatility are probably the same as realized volatility. However, as aforementioned, the spillovers between implied volatility indices is of more practical importance to practitioners because investors can actually gain direct exposure to implied volatility through the use of options and index futures.

Aboura (2003) was the first to study implied equity volatility spillover across international markets. He used volatility indices from the US, France, and Germany and reduced-form vector autoregression to show that there is spillover

⁴ Exposure to realized volatility can be obtained through trading a variance swap, which is an OTC product.

between all three markets, but most significantly from the US to France and Germany. The similarity of this result to those presented above suggests that the drivers of implied volatility spillover are comparable to realized volatility spillover, which is relatively unsurprising.

After Aboura (2003), there was little research on implied volatility spillover because there were few published volatility indices. Following the creation of an implied volatility index for India's largest equity index, Narwal, Sheera, and Mittal (2013) showed that there was a high level of correlation between volatility indices in India, the US, France, Germany, and Switzerland. They found evidence spillover from India to the other markets, which is surprising given that there has been little documented evidence of realized volatility spillover from an emerging market to a developed market. However, this spillover could be driven by a difference in trading hours, which is considered in this paper. Ding, Huang, and Pu (2014) used volatility indices from developed countries to examine if there was any change in the correlation structure following the financial crisis. They found no significant change, but found that the implied volatility spillovers during the crisis became one-dimensional from the US, like what has been documented for realized volatility spillovers.

The three papers mentioned that research implied volatility transmission have been limited to the use of a reduced-form VAR model. However, examining spillover using a reduced-form VAR model is difficult because the estimated coefficients do not have a clear economic meaning. This paper estimates a reduced-form VAR like the previous literature, but then goes further and attempts to identify a structural VAR model. The key advantage of a structural VAR is that by orthogonalizing the residuals across equations, forecast error variance decompositions and historical decompositions can be performed to examine implied volatility spillover at different time horizons.

The method of structural VAR identification used in this paper is motivated by Bessler and Yang (2003), who examined the presence of cointegration in equity indices in different countries. By using a DAG based on the residuals of a vector error-correction model, they identified the contemporaneous correlation structure between these nine indices. They found that Japan is the most exogenous market and explains surprising little about other markets. They also found that the US is significantly influenced by Hong Kong and the UK in the short run, but at a one-month time horizon the US has the strongest impact on price movements. This paper's use of a DAG to identify the structural VAR is also motivated by the

methodology of Yang and Zhou (2013), who used DAG-based structural vector autoregression on credit spreads to examine credit risk spillover during the financial crisis.

3. Data Description

3.1 Data Collection

To examine international implied volatility spillover, I used fifteen implied volatility indices based on equity indices in different countries, shown in Table 1. This list of volatility indices represents every volatility index in the world that is calculated according to a particular methodology described below. Table 1 shows a list of these volatility indices, their underlying equity indices, and the countries or regions that is represented by each index. The column titled “Inception” of Table 1 shows the inception date of each index. These volatility indices are calculated⁵ based on the prices of out-of-the-money puts and calls on the underlying equity index, weighted to maintain a constant volatility⁶ exposure, and represent the fair strike of a one-month variance swap on the underlying equity index. Intuitively, the level of a volatility index at a given point in time represents the market’s consensus of what the volatility of the underlying equity index will be over the next month, in annualized terms.⁷ For example, if the level of the VIX is 10, the market expects the annualized standard deviation of S&P500 returns over the next month to be 10%.

For each volatility index in Table 1, I collected daily closing values from Bloomberg for two different sample periods: January 1st, 2004 to September 27th, 2017 and March 11th, 2011 to September 27th, 2017. The first sample period was chosen to ensure that all four major US volatility indices were included in the sample and to include the 2008 financial crisis. The second sample period was chosen because it is the largest possible sample period that contains all the volatility indices in Table 1. All indices that were not created by January 2004 are not

⁵ The calculation of these indices is complicated and requires a strong understanding of options theory. For details on the calculation see <https://www.cboe.com/micro/vix/vixwhite.pdf>. For a theoretical discussion of the pricing of a variance swap, see Derman et al. (1999).

⁶ This is crucial to the calculation of these volatility indices because it ensures that changes in the index are not driven by changes in the underlying equity index, but rather to changes in the implied volatility of the equity index. See Derman et al. (1999) for a discussion.

⁷ This is actually not quite true. Implied volatility contains a forward-looking element, but it also contains a volatility risk premium such that there is no arbitrage. This volatility risk premium, which has been shown to be negative (Bakshi and Kapida (2003)), will be ignored for the purposes of this paper.

included in the 2004-2017 sample period, which means that only twelve of the fifteen indices are included in this sample period. The final two columns of Table 1 specify explicitly which indices are in each sample period. A daily observation period is chosen because volatility indices move rapidly⁸, therefore a daily frequency is needed to more accurately describe the volatility spillover effects between countries.

Because different countries have different trading holidays, the data collected from Bloomberg for each volatility index have a different number of observations. To address this, for both sample periods, I found a list of dates that represented each day on which I had data on one or more index. With this list of dates, I merged data for all the indices into one dataset for each of the two sample periods. The resulting datasets had some missing values because not all indices were traded on every day. To fill these missing values, I interpolated according to the Catmull-Rom Spline procedure⁹ separately over the two sample periods. This procedure was chosen because it fills the missing data values according to multiple surrounding data points, which is desired because volatility indices are highly autocorrelated over time.¹⁰

3.2 Dataset Limitations

The first limitation of my dataset is that I was required to interpolate for missing values, as mentioned above. However, I do not believe this interpolation affects the validity of my results for two reasons. First, the number of interpolated values is less than 3% of the total number of observations in both sample periods, which I do not believe is sufficiently large to cause concern. Second, I believe interpolation is theoretically justified. Although an index may not trade on a given day, there are still changes in the markets consensus of 1-month future volatility. By interpolating based on surrounding values, I make an attempt to capture these changes in the market's expectation. Some past researchers have accounted for this issue by dropping all dates on which there is a missing value for one or more index. This is

⁸ For reference, the annualized standard deviation of volatility indices is roughly 10 times that of equity indices.

⁹ For a detailed description of this procedure see

<http://www.eviews.com/help/helpintro.html#page/content/series-Interpolate.html>.

¹⁰ This technique interpolates purely in the time-series dimension. Another potentially better alternative is to use a technique that interpolates across the cross-section, but given the small number of missing values I do not believe that these techniques provided sufficient benefit to account for their added complexity.

a poor solution given that it results in a lot of lost data (over 12% of the total number of observations for both samples in my case).

The second limitation of my data set is that each daily observation of my cross-section of volatility indices does not occur at the same point in time. This is because different countries have different trading hours, so not all closing values for the same date actually occur at the same time. This is a significant limitation, but in my econometric analysis I attempt to impose restrictions in my DAG analysis that control for this. These restrictions are discussed in Section 4.4 and are similar to those proposed in Bessler and Yang (2003).

4. Empirical Framework

This section discusses the DAG-based structural vector autoregression (VAR) methodology this paper uses to study implied volatility spillover. There are three distinct steps in this empirical framework: the estimation of a reduced-form VAR model, the use of a directed acyclic graph (DAG) for identification, and the estimation of a structural VAR based on the identification structure determined by the DAG. The description of this procedure is mostly theoretical because it is a relatively uncommon empirical framework that can be used with any group of stationary time-series. Therefore, this section attempts to serve as a guide for the application of this data-driven solution to the identification problem for any group of time series. Sections 4.2 and 4.4 are parts of this methodology that are relevant to its application with the volatility indices used in this paper.

4.1 Estimation of a Reduced-form VAR

Let Y_t be a $k \times 1$ vector of the values at time t of k covariance stationary time-series. Stationarity in the first two moments of all the time-series contained in Y_t is necessary for VAR estimation results to be valid. Often, first-differencing is required to remove the presence of a unit root. I assume that all of the elements of Y_t have already been transformed into their stationary representations so a reduced-form $VAR(p)$ can be estimated, where p represents the chosen lag length. The model specification is shown in equation (1):

$$(1) Y_t = \delta + \sum_{i=1}^p Y_{t-i} \Theta_i + \varepsilon_t$$

where δ is a $k \times 1$ vector of constants, Θ_i is a $k \times k$ matrix of coefficient estimates on lagged values of Y_t , and ε_t is a $k \times 1$ vector of white noise terms. This model requires an assumption of statistical independence between Y_t and ε_t for elements of the same index in both vectors. Correlation between elements in ε_t cross-

sectionally is permitted. Each of the p matrices, Θ_i , in this $VAR(p)$ model can be estimated via rolling-OLS, assuming the well-known Gauss-Markov assumptions are satisfied for all k equations.

An important condition that has been overlooked in the mentioned previous research that have estimated a reduced-form VAR for implied volatility indices is that autocorrelation of each of the k elements in ε_t makes statistical inference in equation (1) invalid.¹¹ The presence of residual autocorrelation can often be removed through increasing p . However, this generally results in increasing p beyond what is recommended by standard information criterion. Despite this loss of efficiency, increasing p is required to remove the presence of residual autocorrelation.

4.2 Specification of Volatility on a Logarithmic Scale

This section relates directly to the volatility indices used in this paper, but not to the general DAG-based structural VAR methodology and is placed here because it relates to the importance of ensuring that a reduced-form VAR does not contain residual autocorrelation.

After obtaining the volatility indices, I first estimated a reduced-form $VAR(p)$ over each sample period, where Y_t was contained all of the first-differenced volatility indices. The reason for first-differencing is that results of an Augmented Dickey-Fuller test suggested the presence of a unit root, and it is generally better to be cautious and first-difference. After estimating the $VAR(p)$, I tested for autocorrelation in the residuals using an LM test and found significant autocorrelation at all lags, regardless of how much I increased p . Given that autocorrelation is often evidence of functional form misspecification, I attempted to find a different stationary representation of these indices.

Eventually, I found¹² that taking the logarithm of the volatility indices (no first-differencing) and then estimating a $VAR(p)$ resulted in no significant residual autocorrelation, according to an LM test. This suggests that volatility indices move relative to each other on a log scale, rather than a level scale. In other words, when a volatility index moves from 10 to 20 it should be considered a bigger move than a move from 100 to 110, which is consistent with intuition. This problem has not

¹¹ This is well-documented in most time-series textbooks, namely that in the presence of autocorrelation in the residuals when there is a lagged value of the dependent variable, OLS estimation will be biased.

¹² Zhuanxin Ding, Ph.D., Analytic Investors LLC, suggested this as a solution.

been addressed in the three previous studies that have used a reduced-form VAR to model volatility indices. For the rest of this paper, when all volatility indices are modeled, they are modeled in log terms for this reason.

4.3 Directed Acyclic Graph Analysis for Identification in TETRAD

After estimating a reduced-form VAR, a directed acyclic graph (DAG) can be used to identify a structural VAR. Identifying and estimating a structural VAR is of interest to a researcher because it makes investigation of the casual structure among the k elements in Y_t possible. In order to identify a structural VAR, the k innovations in ε_t must be orthogonalized. A DAG provides a method to orthogonalize these shocks in the reduced-form VAR.

The DAG technique is a relatively recent advance in causality analysis that was originated by Pearl (2000) and Spirtes et al. (2000). The goal of a DAG is to produce a picture representing the causal flow among variables in contemporaneous time. This is done using an algorithm called the PC algorithm, which I attempt to describe intuitively.¹³ The PC algorithm is implemented in this paper using TETRAD¹⁴, which was also used by Bessler and Yang (2003) and Yang and Zhou (2013). In the interest in assisting further researchers with this technique, the discussion that follows includes a brief description of how to implement DAG analysis in TETRAD.

As an example, assume that I am interested in the causal relationship between three time-series: A, B, and C, so I put their correlation matrix into TETRAD as a data box. To produce a DAG graph in TETRAD, I connect this data box to a search box and choose to use the PC algorithm. This search box will then produce a DAG graph, based on the PC algorithm. An example of a DAG graph that could be produced is shown in Figure 1. This example shows the three types of causal relationships¹⁵ that can be found in a DAG. The first is the directed arrow from A to B, which means that the PC algorithm found the following causal relationship in contemporaneous time: *A causes B*. The second type of relationship that the algorithm can find is the undirected arrow, which is shown in Figure 1 between B and C. This represents the following causal relationship: *B and C are*

¹³ Readers interested in the technical details should consult Spirtes et al. (2000).

¹⁴ A TETRAD manual and installation can be found at <http://www.phil.cmu.edu/tetrad/>.

¹⁵ When there is a larger number of variables, the causal relationships become more complex. However, the details are not necessary for the purpose of this paper and interested readers should consult the TETRAD manual.

related, but the direction of contemporaneous causality is unclear. The final type of relationship is shown in Figure 1, between A and C. No arrow between the two variables signals that the algorithm found *no contemporaneous association between A and C*.

The PC algorithm determines which of these three causal relationships exists between each pair of variables in the following way. First, it forms a graph with A, B, and C that consists of solely undirected arrows. Next, it removes any undirected arrow if the correlation between the two variables is not statistically different than zero.¹⁶ Variables with remaining undirected arrows are then checked for first order partial correlation, which is the correlation between two variables based on a third variable. If any of these first order partial correlations are not statistically different than zero, the undirected arrows are removed. The algorithm continues this process, checking up to the (number of variables minus two)-order partial correlation.

The remaining undirected arrows are then converted to directed arrows based on considering variables in triples.¹⁷ In my example, the undirected arrow between A and C was removed through the algorithm's iterative process described above. Therefore, I know that any correlation between A and B cannot come from C and any correlation between B and C cannot come from A. Lastly, this means I can direct the arrows from A to B and C to B. This process is called the *notion of supset*. However, looking at Figure 1, I see an undirected edge between B and C, which contradicts the directed arrow from B to C I found according to above *notion of supset*. The algorithm produces an undirected arrow in this case when the *notion of supset* combined with the calculated correlations in iterative process yield a possibility of the arrow being directed either direction. For a more detailed discussion of how this can happen, see Bessler and Yang (2003).

Combining the iterative process of testing unconditional and conditional correlations and the *notion of supset*, a DAG can be produced like the one in Figure 1 based on the correlation matrix of any number of time-series. For all the DAG's produced in this paper, the PC algorithm is exactly as described above, and it is implemented in TETRAD. The PC algorithm test statistic used in this paper is

¹⁶ There are multiple available correlation tests available in TETRAD.

¹⁷ Since our example only has three variables, this is only done once. In the case of more than three variables, this procedure is performed in all possible combinations of three variables.

Fisher's Z-statistic¹⁸, and significance is tested at the 5% level¹⁹. To implement this procedure in TETRAD, first input the correlation matrix in a data box. Next, connect the data box to a search box and specify the desired test and significance level. Lastly, connect the search box to a graph box and choose "Make bidirected edges undirected".

4.4 Including Prior Knowledge in a DAG to Deal with Different Trading Hours

When producing a DAG, the researcher can also incorporate prior knowledge about casual relationships between the variables. This can be done in TETRAD on the "Knowledge" tab of the Search box by explicitly requiring or restricting directed arrows between certain variables. This prior knowledge can come from a variety of sources and is not required. The PC algorithm works the same way, except it skips the steps to produce the arrows that the user has either required or forbidden.

As discussed in Section 3.2, one of the limitations of the data used in this paper is that markets in different countries are open at different times. The trading hours (EST) of the different countries' volatility indices used in this paper is shown in Table 2. Bessler and Yang (2003) suggested imposing prior knowledge to deal with this problem of non-synchrony and using the following restriction: *An index from country A cannot cause an index from country B if country B's trading is closed before country A's trading opens*. The consequence of this restriction is that North American and European markets cannot cause Asia-Pacific markets in contemporaneous time. This makes sense intuitively because it is unfair to allow one market to cause another when the markets are not even trading at the same time. In all of the DAGs produced in this paper, this same restriction is used.

4.5 Estimation of a Structural VAR

The final step of this methodology is to orthogonalize the k elements in ε_t from equation (1) to estimate a structural VAR. This can be done in multiple ways, and in this paper, it will be done through imposing restrictions on the relationships among the k elements of ε_t , based on a DAG produced according to Section 4.3. It is well known that for a structural VAR to be identified based on a reduced-form VAR specified as equation (1), there must be $k(k - 1)/2$ restrictions imposed on the k^2 possible relationships between the k elements in ε_t .

¹⁸ This is the test recommended by Bessler and Yang (2003).

¹⁹ Per the discussion in Spirtes et al. (2000) and Pearl (2000), this paper decides to conduct all DAG tests at the 5% significance level.

Let S be a $k \times k$ matrix, where $S_{i,j}$ represents the restriction imposed on the contemporaneous causal relationship from the j -th element to i -th element of ε_t . For all i and j , $S_{i,j}$ will either be NA or 0 . NA indicates that no restriction is imposed on the contemporaneous causal relationship from the j -th element to i -th element of ε_t . 0 indicates that the restriction imposed is the j -th element does not cause i -th element of ε_t , in contemporaneous time.

A DAG provides a perfect way to determine the restrictions contained in S . Given the reduced-form VAR in equation (1), I can produce a DAG based on the correlation matrix of the ε_t . Then, I can populate the matrix S as follows: $S_{i,j}$ is NA if the DAG has a *directed* arrow going from the j -th element to i -th element of ε_t and 0 otherwise. Given how difficult it is for the DAG to produce a directed arrow according to the procedure described in Section 4.3, the DAG can easily identify more than the $k(k-1)/2$ restrictions needed for the structural VAR to be identified.

After the matrix S has been populated, a structural VAR can be with the restriction in equation (2):

$$(2) S\varepsilon_t = v_t$$

where v_t is a vector of k uncorrelated innovations that represents the residuals in the structural VAR. This is commonly known as the “Short-Run Identifying Restriction” and can be imposed through most statistical software. It is important to ensure that restrictions are applied to ε_t , so they only affect the relationships between the variables in the structural VAR in contemporaneous time.

With the structural VAR identified and estimated from the reduced-form VAR with the restrictions in S^{20} , forecast error variance decompositions and historical variance decompositions can be produced since the k elements of v_t are uncorrelated. These decompositions provide a way to examine the effect of a shock to one element Y_t on the other $k-1$ elements of Y_t , at different time horizons.

5. Spillover among Volatility Indices during 2011-2017 Sample Period

The first time period for which I examine spillover among the fifteen volatility indices in Table 1 is from 2011 to 2017. This sample period is chosen because it is the longest sample period that contains all the indices. Per the discussion in Section 4.2, all the volatility indices are modeled in log terms.

²⁰ The matrix algebra of how this is done can be found in any textbook covering VAR analysis.

To reduce dimensionality, I applied principal component analysis on the four volatility series from the US (VIX, VXN, VXD, and RVX). The first component of these series explains over 96% of their total variance, so I used this principal component to represent US implied volatility. A plot of this series is shown in Figure 2.

With the first principal component of US volatility and the other 11 volatility indices, I estimated a reduced-form $VAR(p)$, in the form of equation (1). Schwarz Information Criterion suggests a lag length of $p = 1$, but I chose to estimate a $VAR(7)$ because this is the most parsimonious model such that an LM test does not detect any significant residual autocorrelation. Given the consequences of autocorrelation on standard inference and its unknown effects on DAG analysis, I believe a $VAR(7)$ is the best model choice, despite its loss of efficiency. Moreover, a lag length of 7 seems economically plausible because it suggests that movements of volatility indices have some dependence on their movements one week ago.

Based on the correlation matrix of the residuals from this $VAR(7)$, I produced a DAG as described in Section 4.3, with the prior knowledge discussed in Section 4.4. This DAG is shown in Figure 3. The general result from Figure 3 is that most volatility originates in Asian markets and then is transmitted to the US and larger European indices, like the V2X. This is surprising because it suggests that the US is not the source of implied transmission, contrary to what was found the seminal work in this area by Hamao, Masulis, and Ng (1990). However, the large dimensionality of the DAG makes it difficult to make general conclusions about volatility spillover. Moreover, with eleven time series, variance decompositions based on this DAG are difficult to interpret because they will have 11^2 elements for each time period.²¹

In order to reduce dimensionality further to investigate the role of the US in geographic volatility transmission, I performed principal components analysis to extract the first principal component of indices (in log terms) by region. Table 3 shows the different clusters of volatility indices, which are decided purely based on geographic region.²² The four clusters are the US, large European indices, small

²¹ These decompositions are available upon request and were not included in this paper because of their size.

²² India volatility and Brazil volatility are not included in a group because they do not have a very high correlation with any of the groups of indices in these clusters, nor do they fit nicely into a group intuitively.

European indices, and Asian-Pacific markets. I broke the European indices up into two categories because I hypothesized that a large European index that covers multiple countries, like the V2X, will likely play a much different role in global volatility transmission than an index that represents only one country, like the VCAC. Within each of these four clusters, I extracted the first principal component. These four principal components are plotted in Figure 4.

With these four principal components, I estimated a reduced-form $VAR(7)$. A lag length of 7 is chosen because it was the best fitting model for all eleven indices, so it most likely describes the data generating process for these four principal components. Moreover, the residuals of this $VAR(7)$ do not have significant autocorrelation, according to an LM test.

Based on the correlation matrix of this $VAR(7)$, I produced the DAG shown in Figure 5 with the prior knowledge discussed in Section 4.3. This DAG confirms the general trend shown in Figure 3, namely that implied volatility spillover originates in Asian markets and is transmitted to US and large European markets, and then further from large European markets towards small European markets. At first, this may be a surprising result that is inconsistent with economic intuition. However, it is important to note that the DAG seeks to identify casual relationships *only* in contemporaneous time because these are the restrictions needed to identify a structural VAR. Spillover from Asia to the US and Europe contemporaneously makes sense, because this is driven by the difference in trading hours. Given that Asia closes before US and Europe open on the same day, spillover from Asia should be expected. These results are supported by the importance that volatility traders place on reading news from Asia prior to the US market open.

With the DAG in Figure 5, I now estimated a structural VAR, as discussed in Section 4.4. Based on this DAG-structural VAR, I produced the forecast error variance decompositions shown in Table 4. The results in Table 4 show that even at a longer time horizon of one month (approximately 20 trading days), shocks to volatility indices in Asia are the dominant factor in explaining the variance of volatility indices in all other regions.²³ Shocks to US indices only explain 15% of the variance of Asian indices at a one-month time horizon, while shocks to Asian indices explain 28% of the variance of US indices at the same time horizon. Moreover, shocks to Asian indices explain about 5 times more of the variance in

²³ Aside from the effect of shocks to one region on itself, which should be the greatest, trivially.

all European indices than shocks to US indices at shorter time horizons of around a week, and about 2 times more than the US at one-month.

The results in Table 4 are surprising because they suggest that Asia plays the largest role in global implied volatility transmission, regardless of the time horizon. The US plays the second largest role, but it is far smaller than Asia's, especially at shorter time horizons. The biggest reason for this short-term effect is likely the difference in trading hours. Volatility traders are generally religious about reading Asian market news prior to the US market open, so it does make sense that Asia leads on a short-term basis. Bessler and Yang (2003) also found this same result, namely that Asia dominates spillover for equity indices in the short term. However, the more interesting puzzle is what causes Asia to dominate the US at longer time horizons. Bessler and Yang found that the role of Asia in equity return spillover decayed quickly over time and the US was the largest driving force at a one-month horizon. Past studies on realized volatility transmission have found similar results, but realized volatility is calculated from returns, and hence, should have the same spillover dynamics as Bessler and Yang found for equity returns. The results of this section suggest that the drivers of implied volatility spillover across regions are different than realized volatility, which would be a good area for further research.

6. Implied Volatility Spillover by Region during the 2007 Financial Crisis

In this section, I examine implied volatility spillover by region during the 2007 global financial crisis, using the twelve volatility indices in Table 1 that are specified in this sample period. Per the discussion in Section 4.2, all volatility indices are modeled in log terms.

Using the clustering of indices by region from Table 3, I performed principal component analysis on each grouping of volatility indices by region. Again, the four clusters are the US, large European indices, small European indices, and Asian markets. The first principal components of each of these clusters are plotted in Figure 6. With these four first principal components, I estimate a reduced-form $VAR(12)$. A lag length of twelve is chosen because it is the shortest lag length such that no statistically significant residual autocorrelation is present, according to an LM test. Although a lag length of twelve is greater than suggested by standard information criterion, I believe this is the best model choice to avoid the unknown consequences of autocorrelation in DAG analysis.

Based on the residual correlation matrix from $VAR(12)$, I produce the DAG shown in Figure 7 using the knowledge specified in Section 4.3. Comparing this DAG with the one produced over a smaller sample period shown in Figure 5 highlights two differences. Asia is still the main source of implied volatility spillover in contemporaneous time, but the spillover over this longer sample goes directly from Asia to smaller European indices, unlike what was found in Figure 7. The second difference is that PC algorithm identifies a relationship between the US and smaller European indices, compared to Figure 5 where no causal relationship was identified. This is consistent with intuition because given that the US played the largest role in the 2008 crisis, it is expected that other indices were more strongly related to the US around that time period.

Using the DAG shown in Figure 7, I now estimate a structural VAR according to the procedure in Section 4.5. Given I am interested in investigating how spillover changed during the crisis, I produce historical decompositions instead of forecast error variance decompositions. Unlike forecast variance decompositions, which describe the average movement in the data, historical decompositions show how much a shock to one variable affects the others at every given point in time.²⁴ These historical decompositions are shown in Figure 8.

The four graphs in Figure 8 show the decompositions of each of the four first principal components by region used in the estimated structural VAR into components representing structural shocks to each series. The graphs in Figure 8 must be interpreted with caution early in the sample because the approximations used for historical decompositions become more accurate as time goes on. This is a well-known problem and the length of time needed for accurate approximations depends on the proximity of dominant root of the VAR process to one. However, I believe that this is not a concern in interpreting these decompositions during the financial crisis because there is over three years of daily data in the sample prior to the crisis.²⁵

Looking at the historical decompositions of all four series during the 2007 financial crisis, the general result is very similar to what was found in Section 5. Volatility was most elevated in all regions around the middle of 2008. Looking at the four graphs in Figure 8, it is evident that most of the increase in implied

²⁴ These historical decompositions are produced in the standard way described in any elementary time-series book, based on the previously estimated DAG-structural VAR.

²⁵ Moreover, the estimated roots of the VAR process are not very close to one.

volatility in all regions during the crisis was driven by shocks to Asian markets.²⁶ Following the big spike in volatility driven by Asia, Figure 8 shows that the sustained periods of high implied volatility, especially in Europe, were driven primarily by shocks in the US. Intuitively, this result is similar to the result from Section 5 in that Asia plays the biggest role in the short term, but the impact of the US increases at longer time horizons. However, this is very different from previous literature, which found that the US is the source of the most spillover during the crisis. These results suggest that the US was an important source of implied volatility during the crisis, but the initial spike was overwhelmingly driven by Asia.

Another interesting result from Figure 8 is found from looking at the end of 2011, when all the regions experienced a rise in implied volatility. In August 2011, the Standard and Poor's downgraded US sovereign debt from AAA to AA+ and global stock markets crashed. The historical decompositions in Figure 8 show that the rise in implied volatility over this time period was almost entirely driven by the shocks in US. US shocks even played a larger role in the rise of Asian implied volatility than Asian shocks themselves. This result is consistent with intuition, since this credit downgrade was an event that originated entirely in the US.

The volatility spillover surrounding the 2011 US credit downgrade identified by the historical decompositions in Figure 8 are significantly less puzzling than the spillover found during the crisis. This suggests that this strange spillover during the crisis could be attributable to a failure of the historical decomposition approximation at this point in the sample period. On the other hand, this result could be coming from the fact that I produced a DAG based on the whole sample, rather than just the period in which I was interested. Therefore, the restrictions imposed to identify the structural VAR could be an inaccurate description of the contemporaneous causal relationship among these regions during the crisis, even if they are an accurate description of this relationship over the whole time period.

In order to determine whether this is the case, I re-estimate the same $VAR(12)$, except over three separate sample periods: 1/1/2004 – 12/31/2006, 1/1/2007 – 12/31/2009, and 1/1/2010 – 9/27/2017. I choose to use the same lag length for all periods because if a $VAR(12)$ best describes the data generating

²⁶ Aside from the effect of US shocks on itself.

process over the whole sample period, it arguably should be the best choice for a model over any subset of this sample period.²⁷

Based on the three residual correlation matrices from each of the *VAR*(12) over the three sample periods, I produce the DAGs shown in Figure 9 using the prior knowledge specified in Section 4.3. Looking at the DAG for the period during the crisis (1/1/2007 – 12/31/2009), it is clear that a change in the contemporaneous causal structure occurred. Notably, spillover from Asia to the US in contemporaneous time was not present during the crisis.

Using the DAG in Figure 9 from the time period including the crisis, I estimate a structural VAR and produce the variance decompositions shown in Table 5. Comparing these decompositions with those in Table 4 shows that the shocks in the US play a much greater role in explaining the variance of implied volatility in other regions during the crisis. Asia still is the dominating factor at a short time horizon, which is likely due to the difference in trading hours as before. On the other hand, at a time horizon of one week or longer the US plays a bigger role than Asia for European indices. The decompositions in Table 5 also show that as most time horizons, the variance of a given volatility index was more effected by shocks across the other regions during the crisis period than it was in a normal period (Table 4). This suggests that implied volatility indices are more interdependent during a time of crisis, which is consistent with what has been demonstrated for realized volatility spillover. Overall, these results are mostly consistent with the past literature on the crisis (Duncan and Kabundi (2013); Dungey and Martin (2007); Karunanayake et al. (2010)), but are distinct in that they show shocks in Asia are still a greater factor than shocks in the US for explaining volatility movements in Europe at shorter time horizons. In addition to Section 5, the results of this section are further evidence that there is a difference in the dynamics of implied and realized volatility spillover that merit further research.

7. The Individual Sources of Implied Volatility Spillover during the 2007 Financial Crisis

This section attempts to extend the results from Section 6 to examine the sources of implied volatility spillover leading up to the crisis, but on an individual rather than regional level. The volatility indices used in the section are the indices that are

²⁷ LM tests at multiple lag lengths over all three time periods fail to detect residual autocorrelation at any reasonable level of significance.

included in the 2004-2017 sample period, as noted in Table 1. Per the discussion in Section 4.2, all volatility indices are modeled in log terms.

To reduce dimensionality to make the DAGs less cluttered, I performed principal component analysis with the four US indices (VIX, VXN, VXD, and RVX) and extract the first principal component to represent US volatility indices. A graph of this first principal component, along with the other 8 volatility indices (in log terms) is shown in Figure 10. Using the nine time series in Figure 10, I estimate a reduced-form $VAR(9)$. A lag length of 9 is chosen to ensure that there is no residual autocorrelation detected by an LM test.

Based on the residual correlation matrix from this $VAR(9)$, I produce the DAG shown in Figure 11 with the prior knowledge specified in Section 4.4. The results from this DAG are consistent with the results I found in Section 5 for the first principal components of each region: in contemporaneous time, most spillover occurs directly from Asia to all other regions. Interestingly, I find that all Asian volatility indices are sources of spillover.

I now estimate a structural VAR based on the DAG in Figure 11 and produce the historical decompositions shown in Figure 12 because I am interested in the relationships among structural shocks to each of the variables over time.²⁸ The first interesting result from Figure 12 is shocks to the VHSI (Hong Kong's volatility index) contributes the most to the increase in implied volatility in other countries. During the 2007 financial crisis, the only index that contributed to the increase of implied volatility in the US (outside of itself) was the VHSI. In Section 6, I find that there is spillover from Asia to US leading into the crisis. The results here show that this spillover was mainly driven by shocks in Hong Kong, while shocks in Japan (VNKY) and Korea (VKOSPI) had little impact on the US. This suggests that this recurring spillover I find from Asia to the US is not purely attributable to a trading hours' difference because if it was, shocks Japan and Korea would have a more similar effect on the US as those in Hong Kong. My finding that Hong Kong is a source of spillover, yet Japan is relatively exogenous, is consistent with Bessler and Yang's (2003) findings, but contradicts the findings of Ding, Huang, and Pu (2014). The similarity of my results with those of Bessler and Yang shows the value of using a structural VAR to examine spillover, instead of the reduced-form VAR used by Ding, Huang, and Pu. By using a reduced-form

²⁸ Figure 12 only contains select historical decompositions because including all 81 decompositions makes interpreting the results difficult. The full historical decompositions are available upon request.

VAR with non-orthogonal innovations, Ding, Huang, and Pu are not able to decompose the movements of each implied volatility index into shocks to other indices and hence, cannot uncover Hong Kong as an initial source of implied volatility spillover.

Looking at the decompositions of volatility indices in London (VFTSE) and in Europe (V2X), I find that again the shocks in Hong Kong drive most of the increase in implied volatility entering the financial crisis, while Japan and Korea play an insignificant role. The rest of the increase in implied volatility in London and Europe is attributable to shocks in the US. If I look at the decompositions of indices in London and Europe relative to each other, I find that London shocks contribute more to European implied volatility than the reverse. Finally, looking at the decomposition of US implied volatility indices into shocks in London and Europe, I find that neither London nor Europe explains much of the increase in US implied volatility during the crisis.

8. Conclusion

Volatility indices, such as the VIX, have become more common as the popularity of equity index options has increased. These indices seek to measure implied annualized standard deviation of the underlying equity index over the next month, based on the implied volatility of the options traded on that equity index. Unlike realized volatility, an investor can gain exposure directly to implied volatility through the use of options, volatility index futures, or an exchange-traded product that tracks the implied volatility index. Despite this greater practical importance of implied volatility, few studies have examined the relationship of implied volatility indices across markets.

This paper uses implied volatility indices from different countries and regions to examine the sources of implied volatility spillover. Although realized volatility spillover has been studied extensively (Hamao, Masulis, and Ng (1990); Engle (1994); Kanas (1998)), to my knowledge only three studies have examined the dynamics of implied volatility spillover (Aboura (2003); Narwal, Sheera, and Mittal (2013); Ding, Huang, and Pu (2014)). This paper expands upon those three papers by using a DAG to identify a structural VAR, which permits the use of forecast error variance and historical decompositions based on the orthogonalized innovations.

In contradiction with the literature on realized volatility spillover, I find that the US is not the dominating source of implied volatility spillover. At shorter time

horizons, shocks in Asia are around five times more influential on other volatility indices than shocks to US volatility indices. Asia's importance in the short-term is most likely due to the difference in trading hours and is consistent with the behavior of volatility traders, who place a large importance on reading Asian news prior to the US market open. The difference in this result from previous literature is likely because few other papers imposed restrictions to account for the difference in trading hours. Furthermore, I show that following the crisis, Asia still plays a larger role than the US at longer time horizons. The difference of this result from realized volatility literature provides strong evidence that implied volatility transmission is not driven by the same factors as realized volatility spillover. A potential robustness check of Asia's spillover dominance would be to use multi-day averages, as done in Yang and Zhou (2017), to account for the difference in trading hours.²⁹

Like previous literature, I find that shocks in the US are a driving factor of heightened implied volatility during the crisis. However, this paper also shows that Asia plays a larger role than the US in the initial spike of implied volatility. Asia's central role in volatility spillover could be due to a trading hours' difference because at the beginning of the crisis information was hitting markets rapidly and other markets may have looked to Asia for insight, prior to their opens. Later in the crisis, I find that any time horizon longer than a few days, the US is overwhelming a driving factor. This result is consistent with previous research. Practically, these findings suggest that volatility traders in Europe and the US can gain valuable information from observing Asia implied volatility movements. Given the current concern over low volatility, traders in the US and Europe should watch Asian markets closely for a potential signal of a forthcoming climb in volatility.

Lastly, I find that the Hong Kong's volatility index (VHSI) accounts for the spillover from Asia during the financial crisis. This result is similar to Bessler and Yang (2003), who found that Hong Kong is a source of equity return spillover during other crises. Surprisingly, there is very little implied volatility transmission from Japan (VNKY) or Korea (VKOSPI), which suggests the factors that influence implied volatility in Asia differ by market. A potential reason for this could be

²⁹ This study was completed prior to the final publication of Yang and Zhou (2017) and the different results found in this paper are likely attributable to their use of two-day averages of first-differenced volatility indices, instead of the log specification used in this paper. Further research is needed to reconcile whether the different results of these two papers is due to the use of two-day averages instead of daily levels or the use of first-differences instead of logs. The discussion in Section 4.2 suggests that the results of Yang and Zhou (2017) might not be robust if they did not remove autocorrelation from the reduced-form VAR.

differences in option trading volume on the HSI, NKY, and KOSPI2 and is a good avenue for further research.³⁰

The findings of this paper highlight the differences between realized and implied volatility spillover. Similar to the role played by Baele (2005) in the realized volatility literature, further research is needed on the economic drivers of implied volatility spillover to understand the reasons for these differences. A reason for this difference could be that implied volatility, unlike realized volatility, contains a risk premium. An avenue for further research would be to extract the volatility risk premium from each implied volatility index and examine the spillover among these risk premia directly. The spillover behavior among the volatility risk premia could explain the differences in spillover results between realized and implied volatility.

Analogous to using an EGARCH model to capture asymmetric effects, an avenue for future research would be examining how implied volatility spillover changes depending on whether it is good or bad news that is being transmitted would be informative. This paper attempts to do this through comparing the periods following the 2008 crisis and after the crisis, but introducing an asymmetry-type term in the VAR framework would be a better methodology. A final avenue for further research would be to examine the profitability of a trading strategy that traded US and European volatility, based on movements in Asia that happened over night.

³⁰ I attempted to examine whether or not this is the case, but to my knowledge the option trading volume on these Asian-Pacific indices over a long period of time is not available.

9. Tables

Table 1 - List of Volatility Indices

<i>Index</i>	<i>Region</i>	<i>Underlier</i>	<i>Underlier Ticker</i>	<i>Inception</i>	<i>In 2011-2017 Sample?</i>	<i>In 2004-2017 Sample?</i>
VIX	USA	S&P 500	SPX	Jan-90	yes	yes
VXN	USA	NASDAQ	NDX	Feb-01	yes	yes
VXD	USA	Dow Jones	INDU	Oct-97	yes	yes
RVX	USA	Russell 2000	RTY	Jan-04	yes	yes
VXFXI	China	iShares China Large-Cap ETF	FXI	Mar-11	yes	no
		iShares MSCI Capped Brazil				
VXEWZ	Brazil	ETF	EWZ	Mar-11	yes	no
V2X	Europe	EUROSTOXX50	SX5E	Jan-99	yes	yes
VFTSE	London	FTSE 100	UKX	Jan-00	yes	yes
VCAC	Paris	CAC 40	CAC	Jan-00	yes	yes
VAEX	Amsterdam	AEX	AEX	Jan-00	yes	yes
	Geneva, Zurich,					
V3X	Basel	Swiss Market Index	SMI	Jun-99	yes	yes
VHSI	Hong Kong	Hang Seng Index	HIS	Jan-01	yes	yes
VNKY	Japan	Nikkei 225	NKY	Jan-01	yes	yes
INVIXN	India	Nifty 50	NIFTY	Nov-07	yes	no
VKOSPI	Korea	Kospi 200	KOSPI2	Jan-03	yes	yes

Table 2 - Opening and Closing Times of Markets in EST

<i>Region</i>	<i>Open Time (EST)</i>	<i>Closing Time (EST)</i>
USA	9:30	16:00
China	18:00	3:00 (+1)
Brazil	7:00	14:30
Europe	3:00	11:30
London	3:00	11:30
Paris	3:00	11:30
Amsterdam	3:00	11:25
Geneva, Zurich,		
Basel	3:00	10:55
Hong Kong	18:00	3:00 (+1)
Japan	19:00	1:00 (+1)
India	10:45	7:00 (+1)
Korea	23:00	7:30 (+1)

Table 3 - Clustering of Volatility Indices by Region

<i>Index</i>	<i>Region</i>	<i>PC Category</i>
VIX	USA	US
VXN	USA	US
VXD	USA	US
RVX	USA	US
VXFXI	China	Asia
		Large
V2X	Europe	Europe
		Large
VFTSE	London	Europe
		Small
VCAC	Paris	Europe
		Small
VAEX	Amsterdam	Europe
	Geneva, Zurich,	Small
V3X	Basel	Europe
VHSI	Hong Kong	Asia
VNKY	Japan	Asia
VKOSPI	Korea	Asia

Table 4 - Forecast Error Variance Decompositions by Region 2011-2017

<i>Variance of Asia volatility indices explained by shocks in other regions</i>				
Day	Asia	Large Europe	Small Europe	United States
1	100.00	0.00	0.00	0.00
2	93.51	0.32	0.00	6.17
5	87.72	0.44	0.00	11.84
10	84.50	1.51	0.10	13.89
20	82.83	2.09	0.09	14.99

<i>Variance of Large Europe volatility indices explained by shocks in other regions</i>				
	Asia	Large Europe	Small Europe	United States
1	23.71	76.29	0.00	0.00
2	30.59	61.91	0.07	7.43
5	33.81	48.29	0.10	17.80
10	35.59	44.22	0.32	19.87
20	41.30	36.70	0.28	21.73

<i>Variance of Small Europe volatility indices explained by shocks in other regions</i>				
	Asia	Large Europe	Small Europe	United States
1	19.30	54.89	25.81	0.00
2	25.85	48.02	20.31	5.82
5	30.84	38.67	15.26	15.23
10	32.04	36.25	14.59	17.12
20	37.26	32.15	12.85	17.75

<i>Variance of US volatility indices explained by shocks in other regions</i>				
	Asia	Large Europe	Small Europe	United States
1	24.18	0.00	0.00	75.82
2	28.32	0.00	0.00	71.68
5	26.11	0.01	0.07	73.81
10	25.81	0.06	0.10	74.03
20	28.41	0.14	0.07	71.38

Table 5 - Forecast Error Variance Decompositions by Region 2007-2009

<i>Variance of Asia volatility indices explained by shocks in other regions</i>				
Day	Asia	Large Europe	Small Europe	United States
1	100.00	0.00	0.00	0.00
2	77.60	13.57	0.02	8.81
5	62.59	17.12	0.16	20.13
10	53.72	22.34	0.32	23.62
20	47.91	24.34	0.25	27.50

<i>Variance of Large Europe volatility indices explained by shocks in other regions</i>				
Day	Asia	Large Europe	Small Europe	United States
1	18.20	81.80	0.00	0.00
2	15.14	75.75	0.06	9.05
5	12.02	67.67	0.21	20.09
10	11.03	64.53	0.31	24.13
20	11.28	57.08	0.25	31.39

<i>Variance of Small Europe volatility indices explained by shocks in other regions</i>				
Day	Asia	Large Europe	Small Europe	United States
1	16.73	66.66	16.61	0.00
2	14.22	67.65	9.21	8.92
5	10.82	63.47	4.11	21.60
10	9.84	62.62	3.56	23.98
20	10.61	56.44	2.58	30.37

<i>Variance of US volatility indices explained by shocks in other regions</i>				
Day	Asia	Large Europe	Small Europe	United States
1	8.28	32.97	0.00	58.75
2	6.93	35.49	0.03	57.55
5	5.14	35.74	0.03	59.09
10	5.17	37.97	0.19	56.67
20	8.00	38.32	0.23	53.46

10. Figures

Figure 1 – Example of possible DAG for A, B, and C

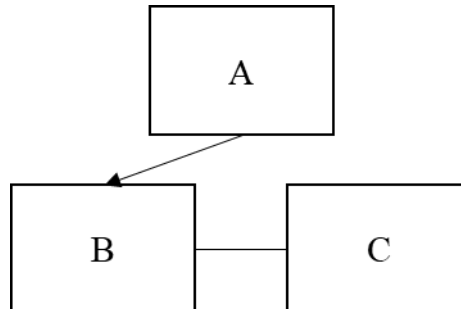


Figure 2 - First Principal Component of US Volatility Indices

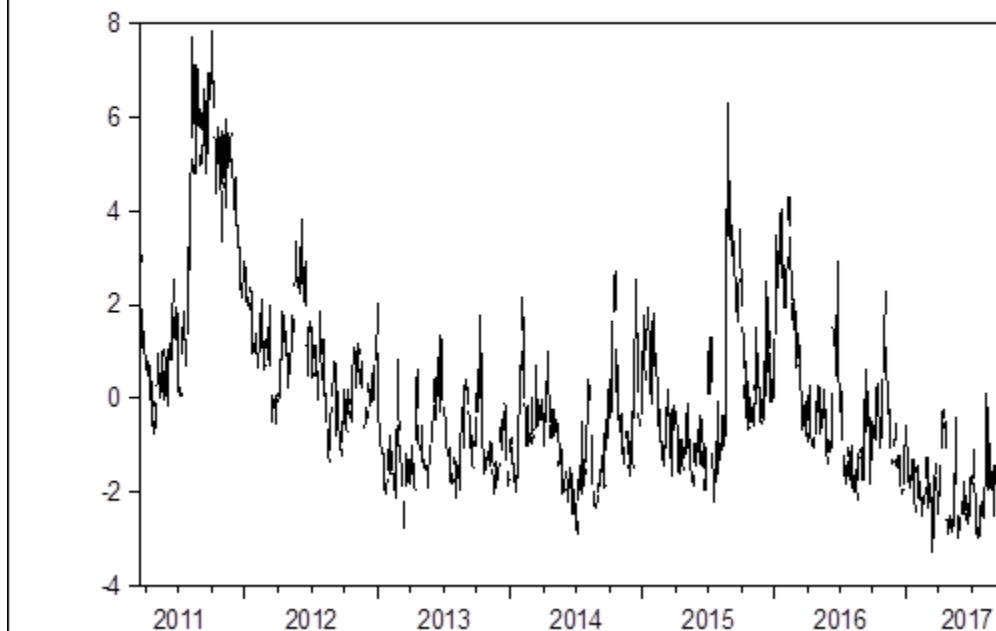
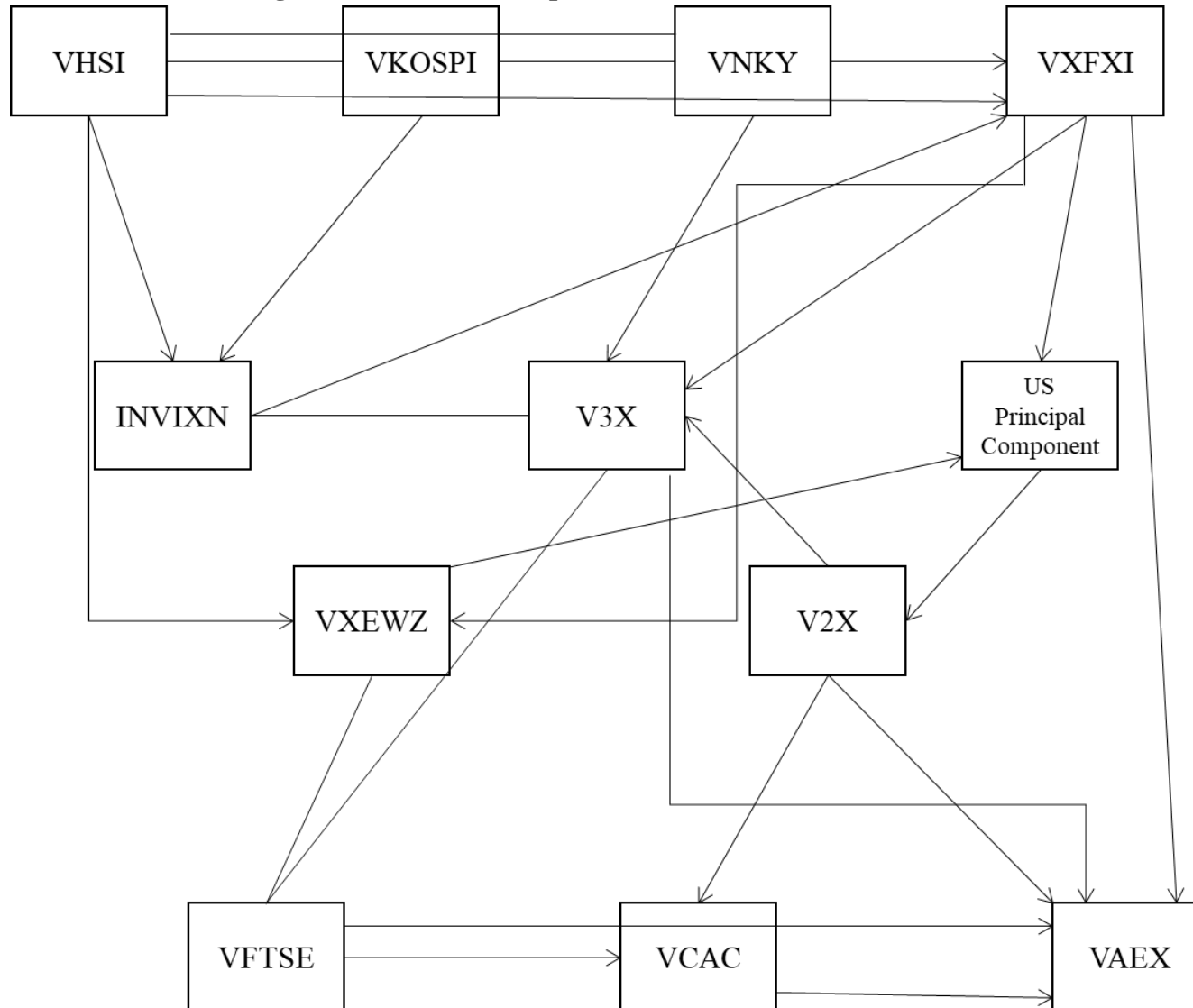


Figure 3 – DAG Contemporaneous Casual Flow 2011-2017



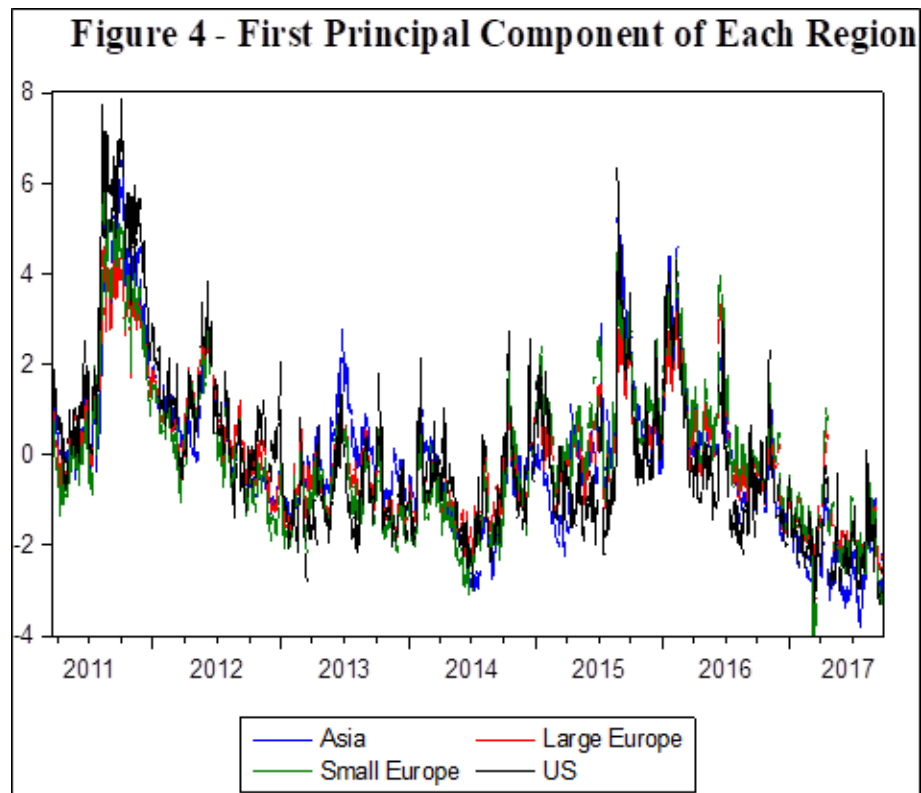
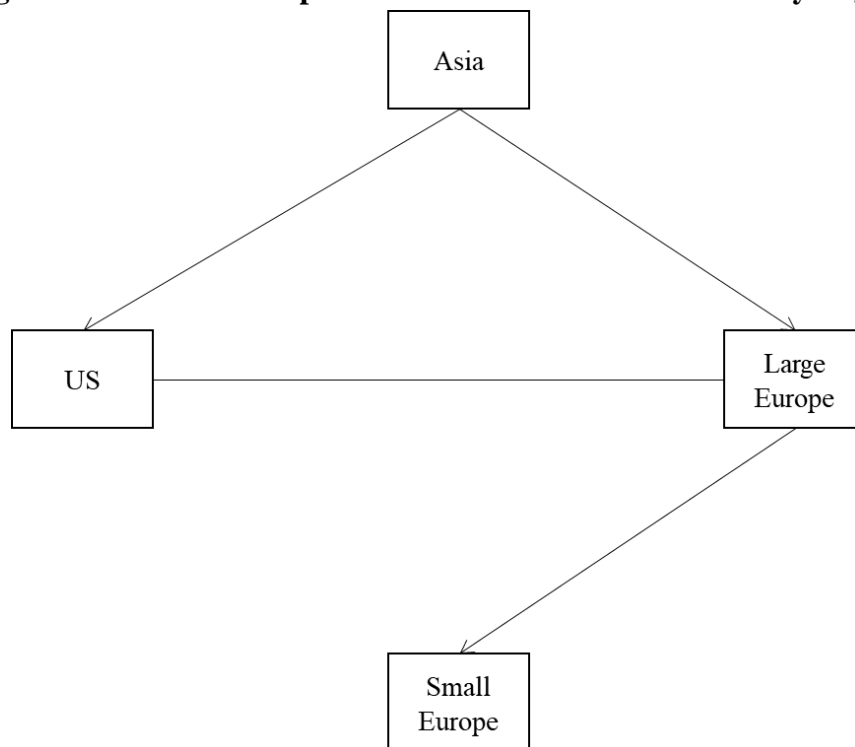


Figure 5 – DAG Contemporaneous Causal Flow 2011-2017 by Region



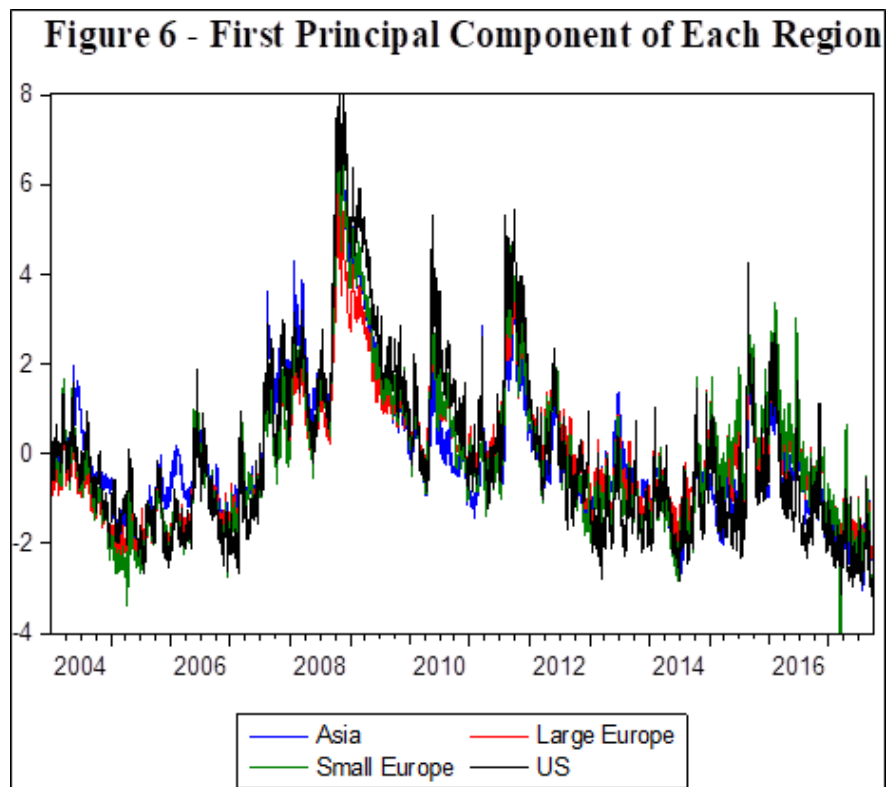


Figure 7 – DAG Contemporaneous Casual Flow 2004-2017 by Region

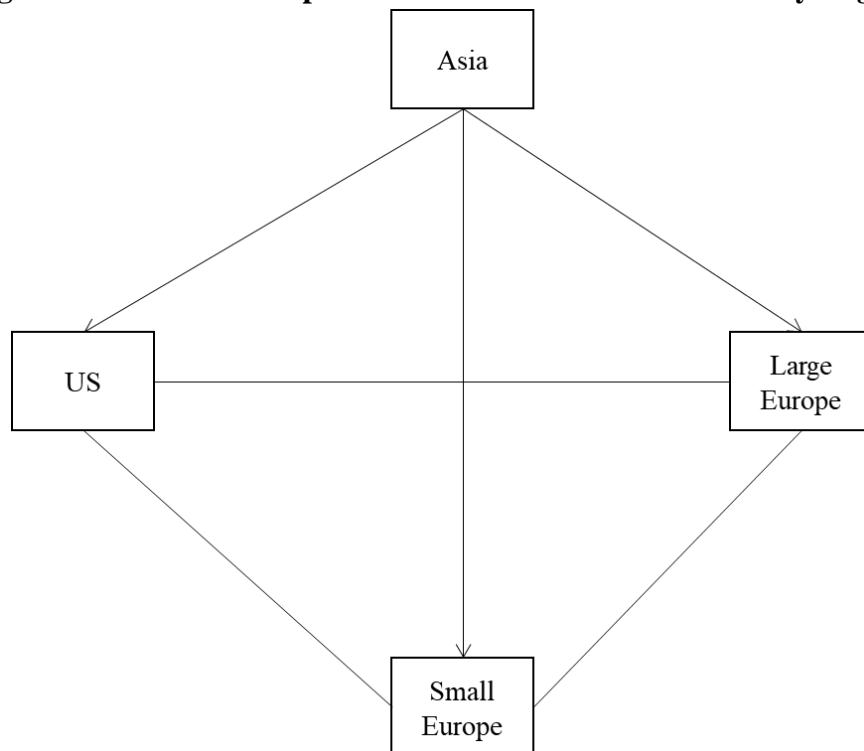


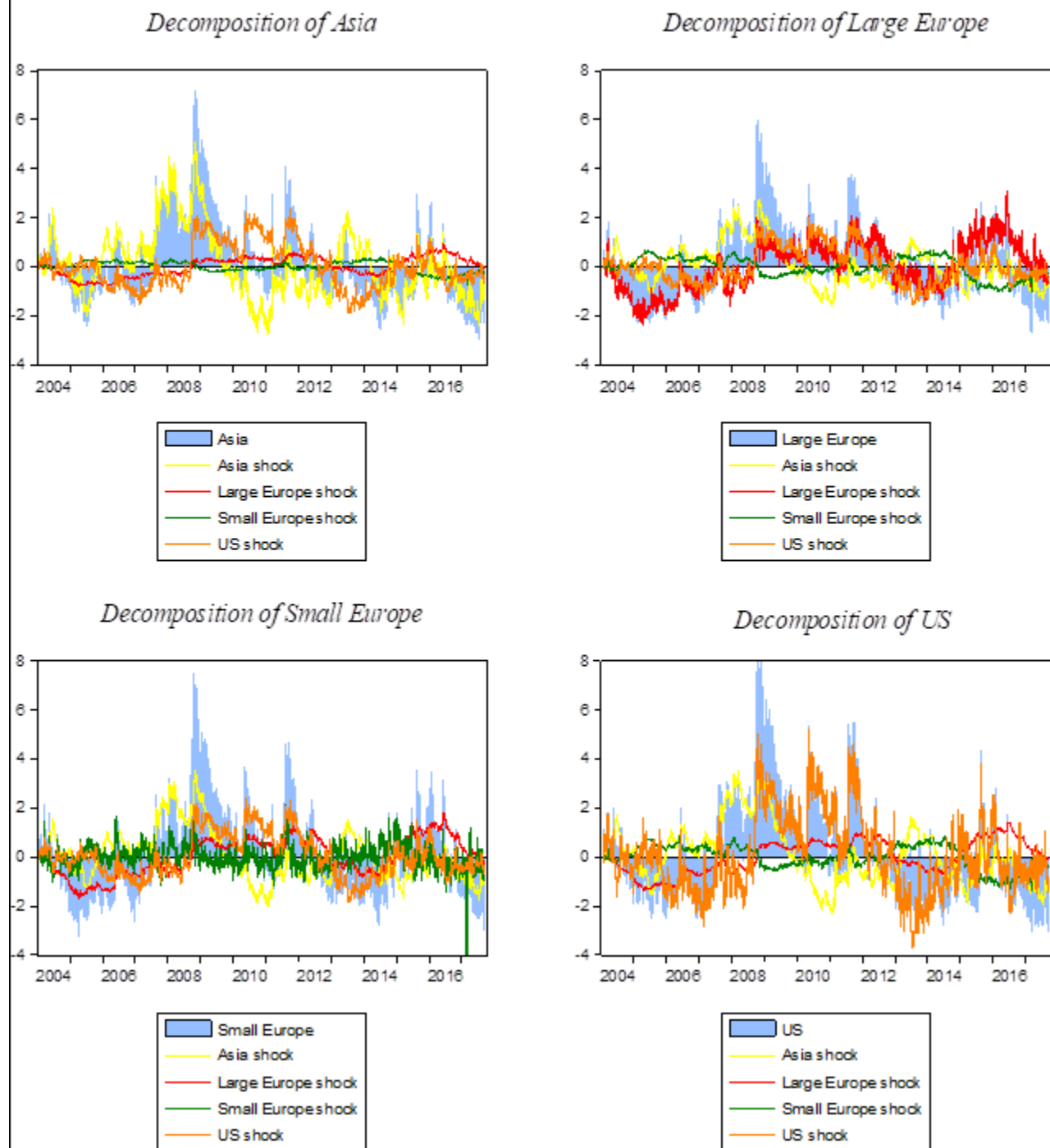
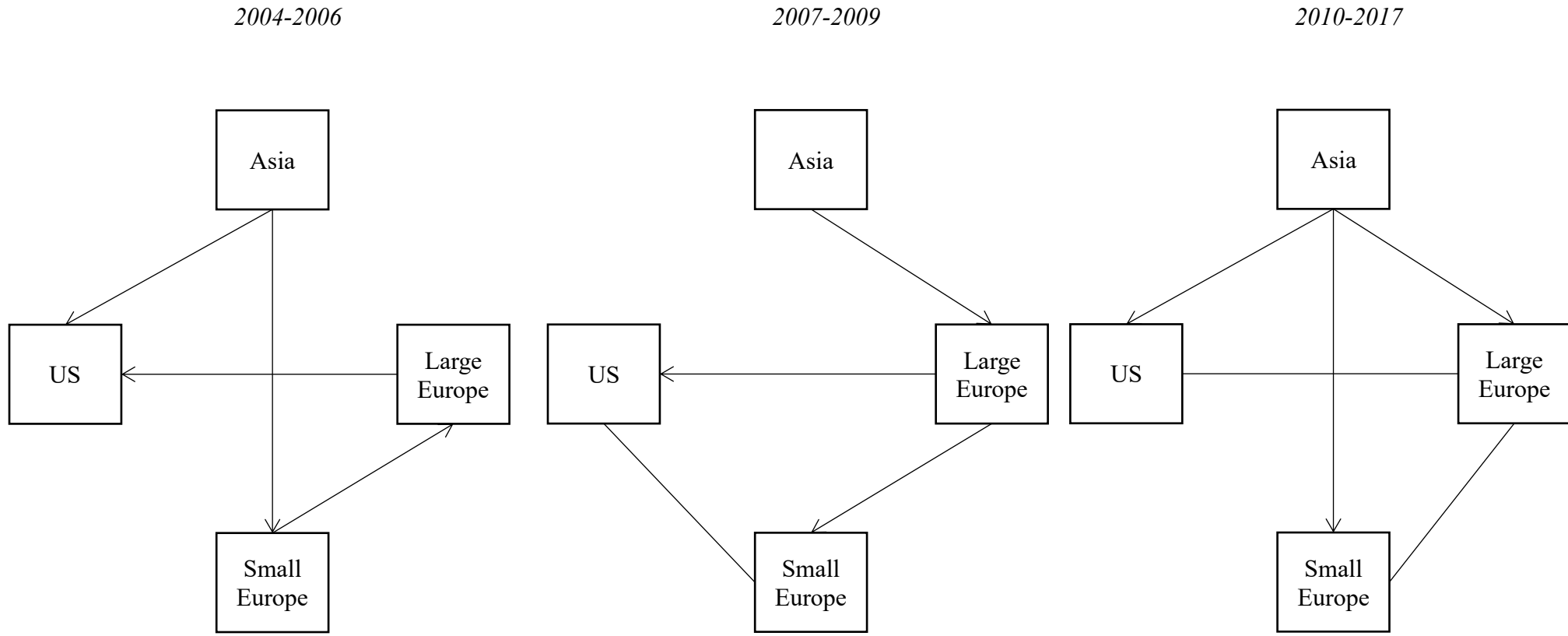
Figure 8 - Historical Decomposition using DAG-based Structural VAR

Figure 9 – DAG Contemporaneous Casual Flows by Regions



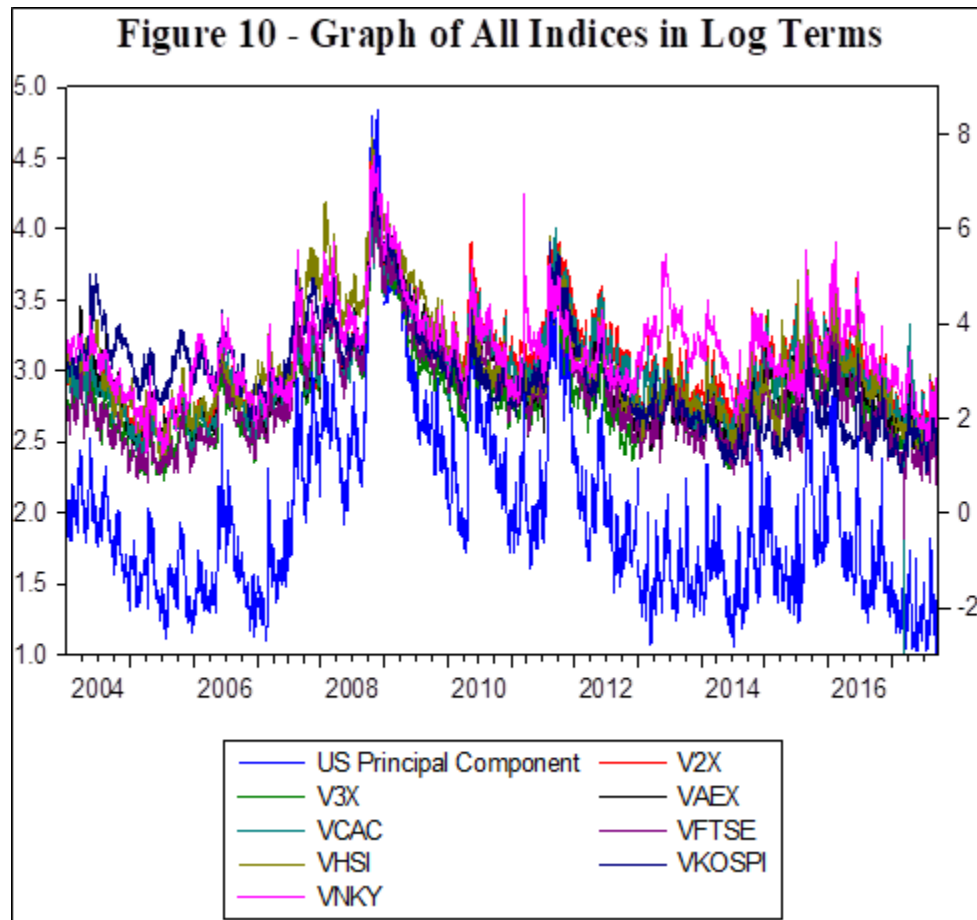


Figure 11 – DAG Contemporaneous Casual Flow 2004-2017

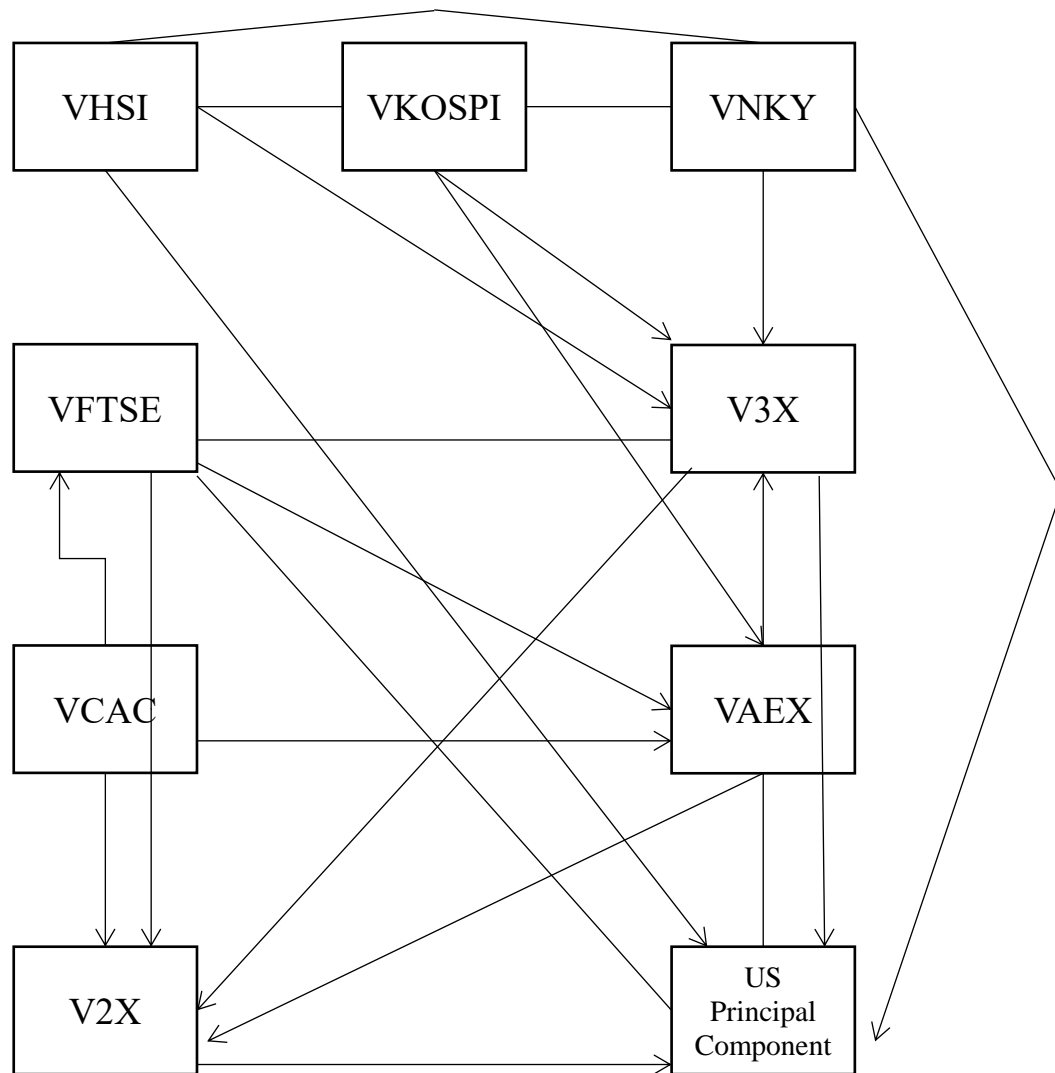
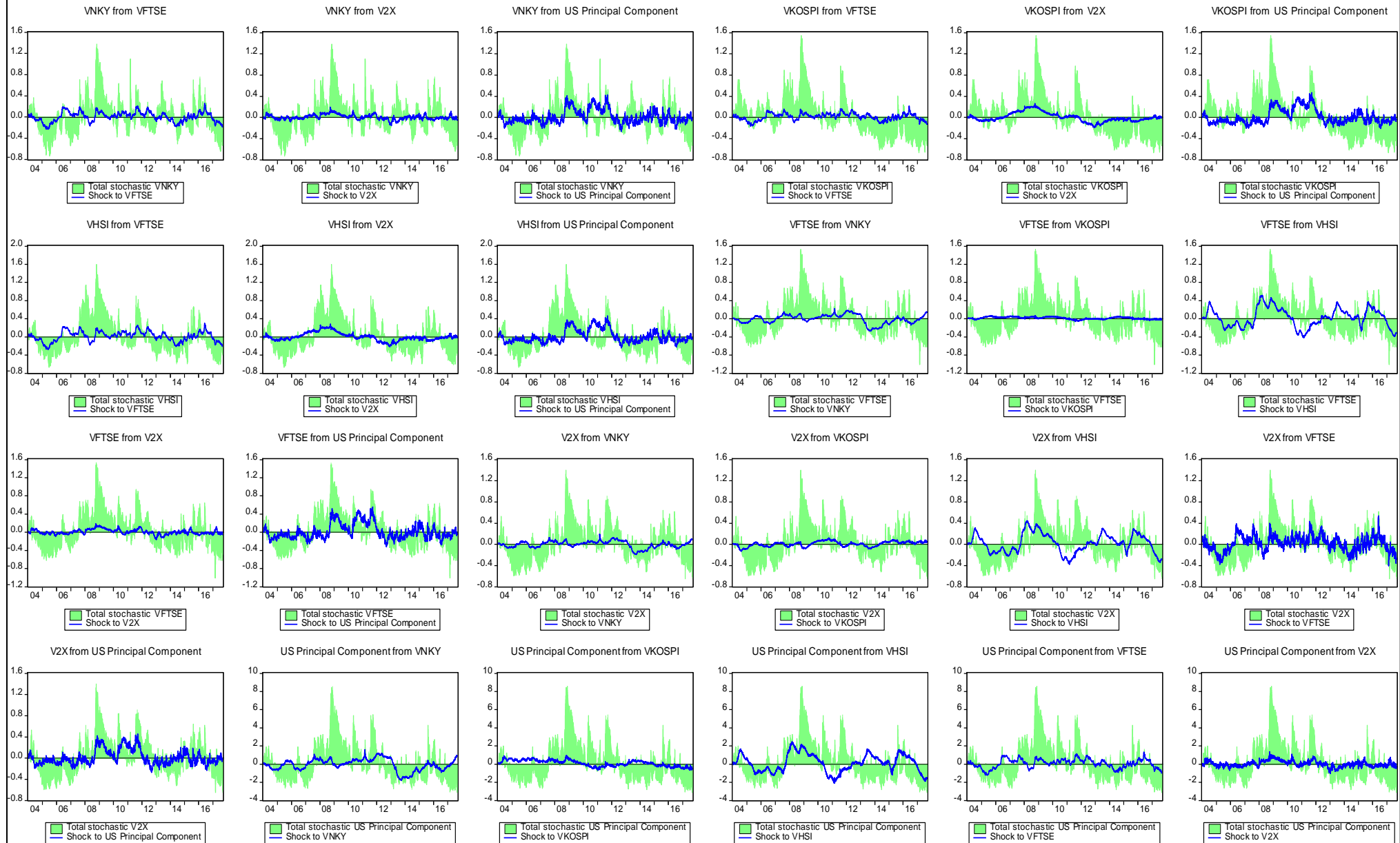


Figure 12 - Selected Historical Decompositions

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