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A Look at Multi-Decompositions of Complete Graphs into Graph Pairs of Order 4

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Multi-decomposition of $K_{2s,t}$ into $2K_2$

Purpose:

Prove the multi-decomposition of $K_{2s,t}$ into $2K_2$.

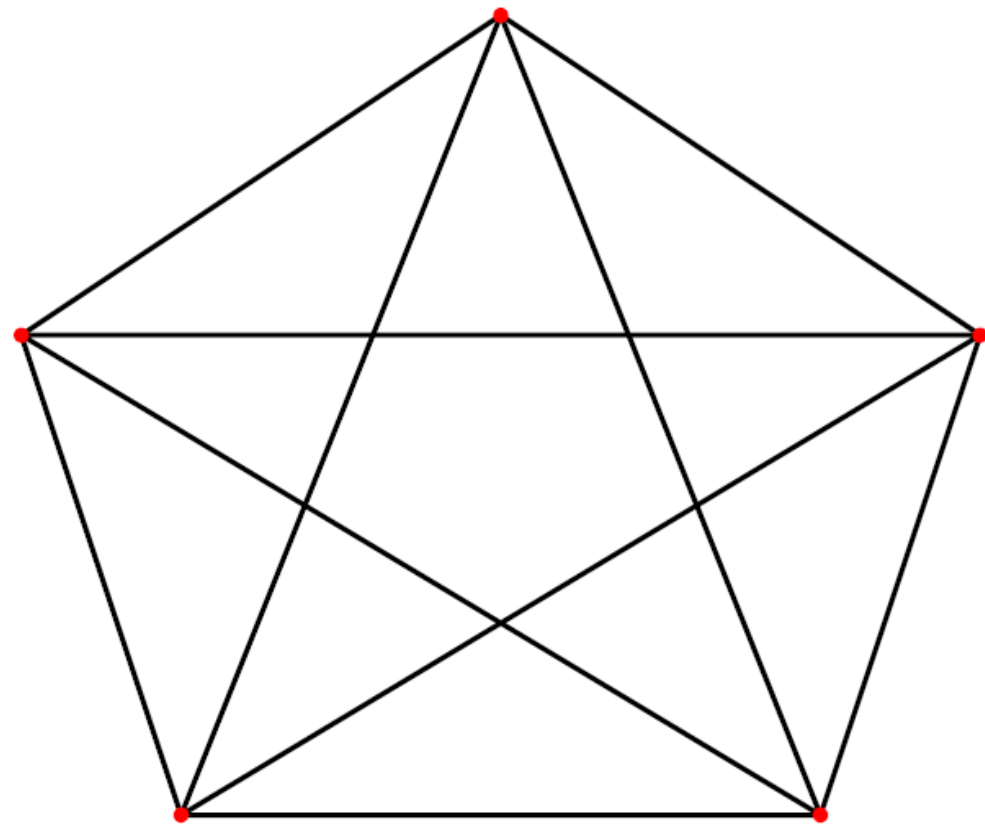
Yizhe Gao
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Instructor: Daniel Roberts

Abueida and Daven did a research paper on the multi-decomposition for graph pairs of Order 4 and 5. They stated that $K_{2s,t}$ can be decomposed into $2K_2$ but they did not verify the statement.

Definition:

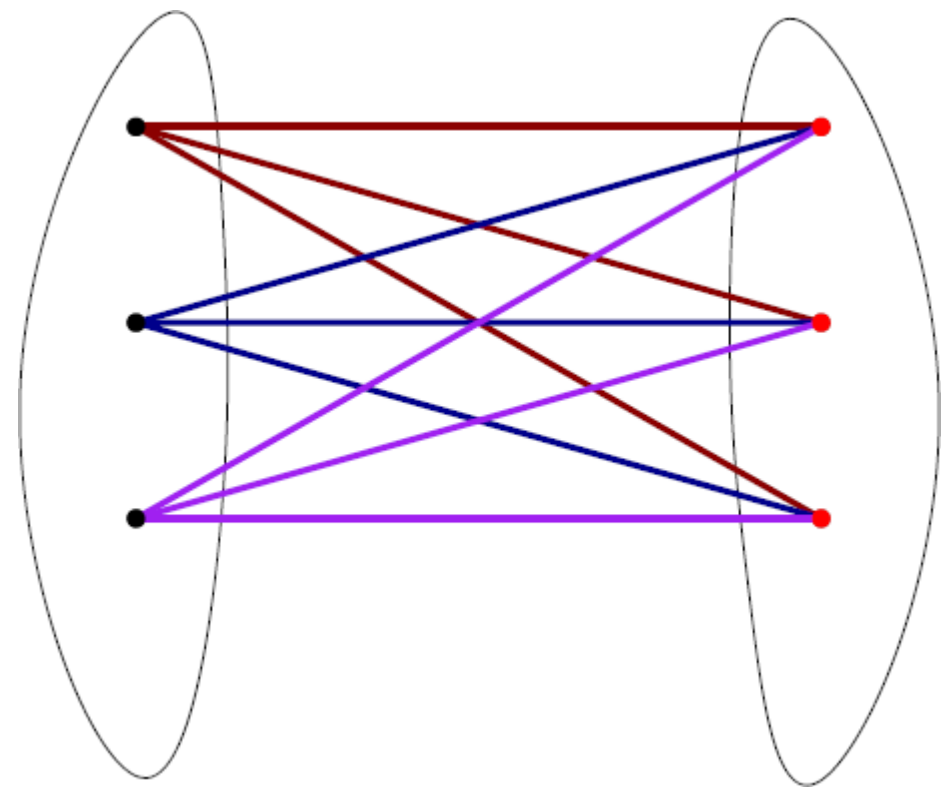
Graph: A graph G is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge two vertices called its end points.

A complete graph is a graph in which each pair of graph vertices is connected by an edge.



K_5

A complete bipartite graph is a graph where the vertices are partitioned into two sets. Every vertex in one part is adjacent to every vertex in the other part.



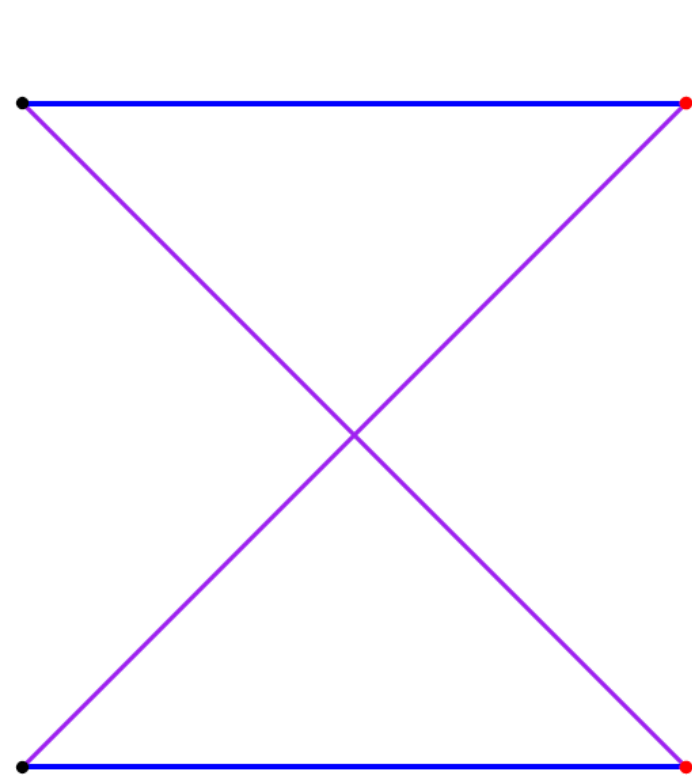
$K_{3,3}$

A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

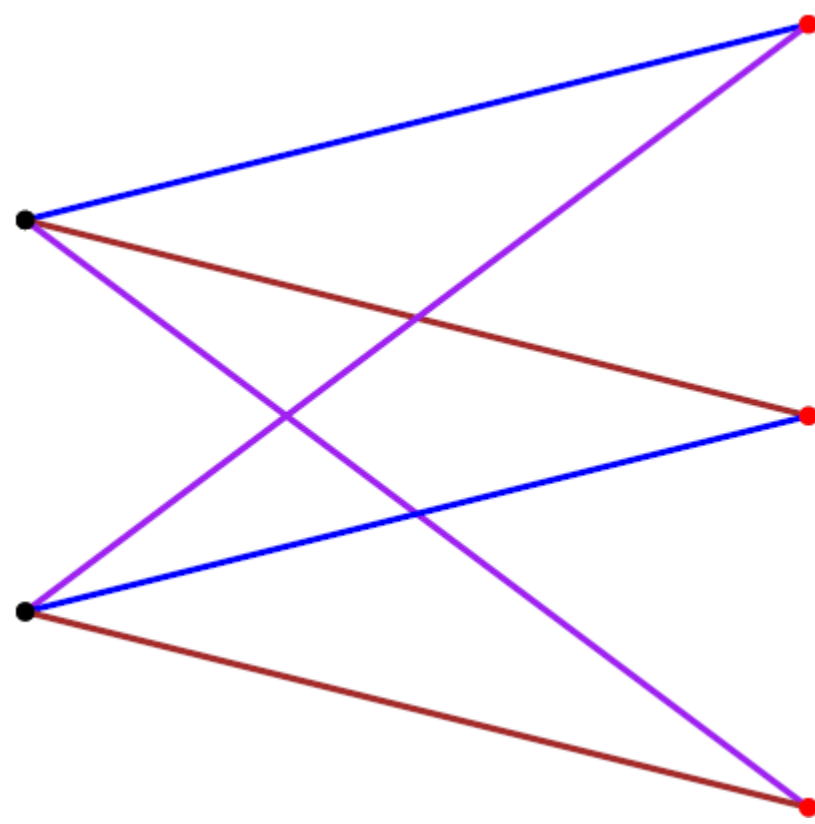
A graph pair of order n is a pair of connected graphs on n vertices with no isolated vertex whose union is K_n .

Procedure:

1. Prove that C_4 divides $K_{2s,2t}$ (see handout)
 2. Prove that $2K_2$ divides $K_{2s,t}$ for $t > 2$.
- Proof: Two constructions that may be used in this proof are listed below.



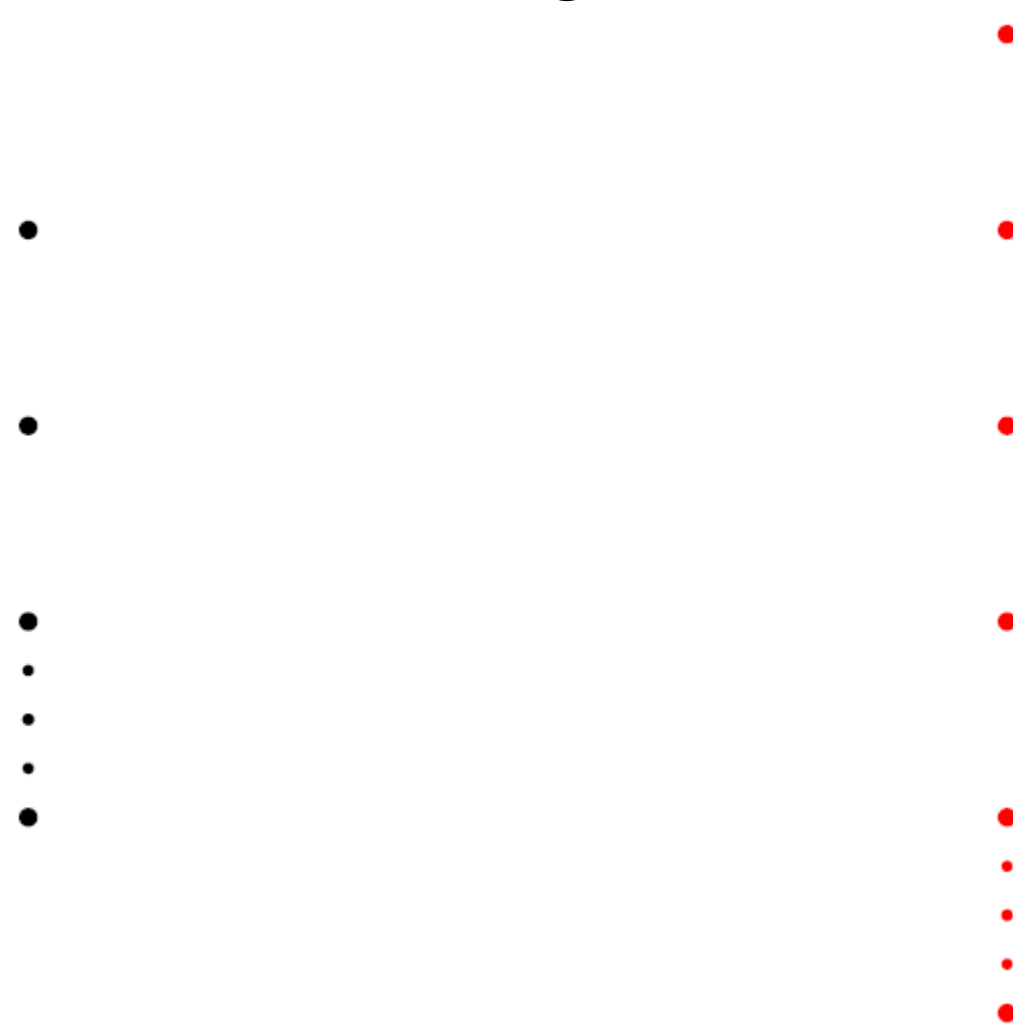
$K_{2,2}$



$K_{2,3}$

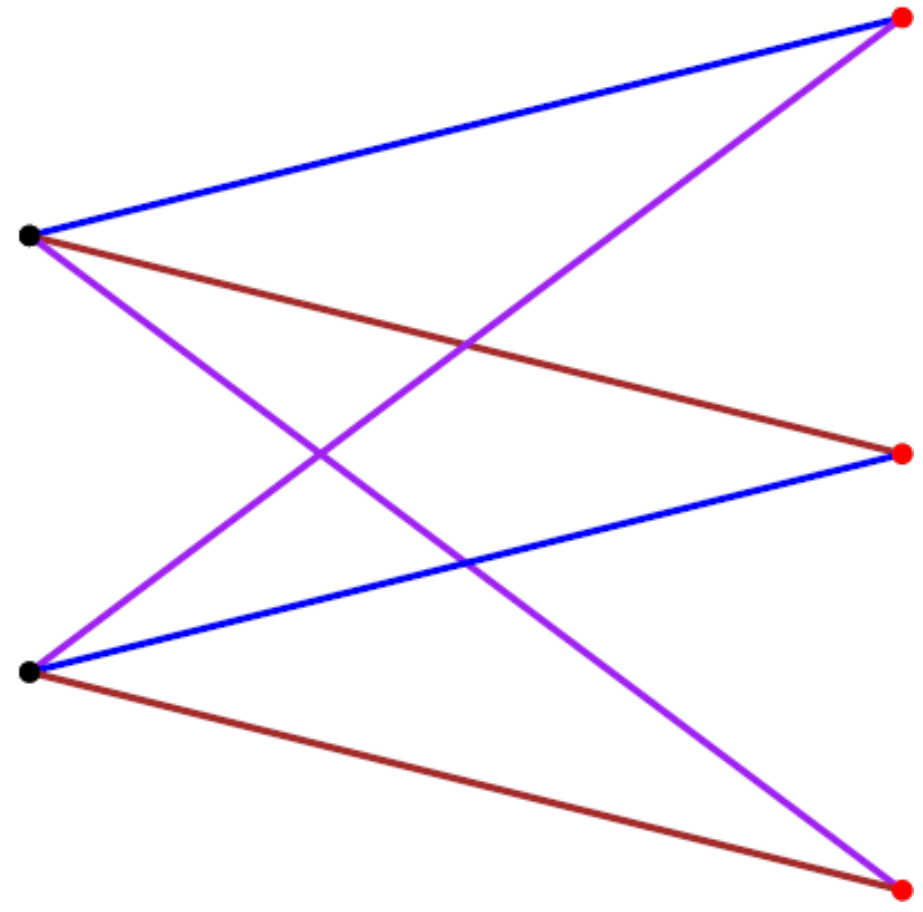
It is easy to prove that both constructions can be decomposed into $2K_2$.

- 1) Assume t is odd. The graph $K_{2s,t}$ is listed below.



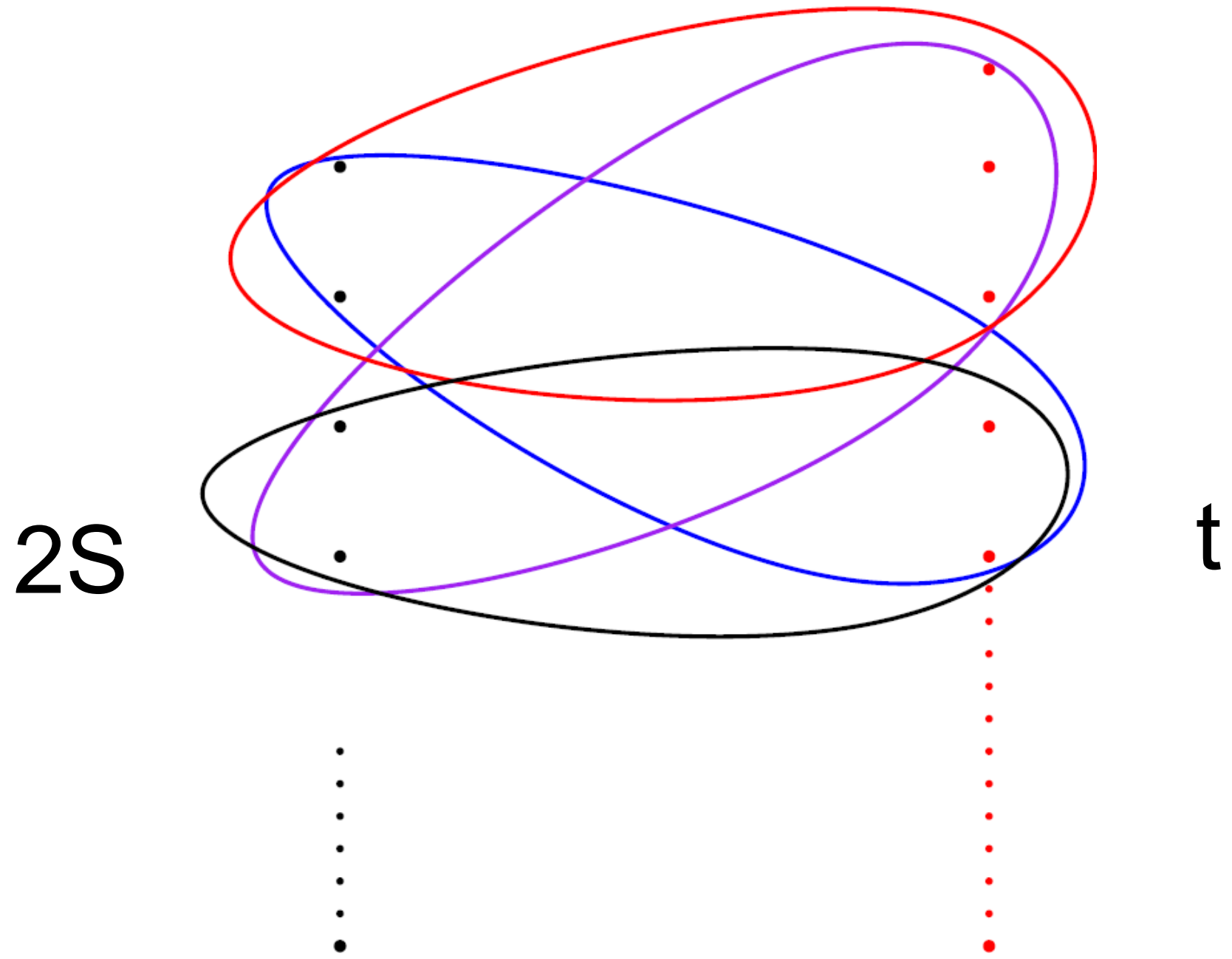
$K_{2s,t}$ when t is odd

When t is 3, $K_{2s,t}$ can be decomposed as follows.



$K_{2,3}$

When $t > 3$, for example, when t is 5, $K_{2s,t}$ can be decomposed into $2K_2$ in following way.



For any $t > 5$, the decomposition will repeat.

- 2) Assume t is even. Then $K_{2s,t}$ can be decomposed into $2K_2$ in a similar way. (The proof is shown in handout)

Results:

For any natural numbers s, t , $2K_2$ divides $K_{2s,t}$

Future study:

Decomposing complete graphs into graph pairs of order 6.