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A Graphical Introduction to Special Relativity Based on a Modern Approach to Minkowski Diagrams

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A GRAPHICAL INTRODUCTION TO SPECIAL RELATIVITY

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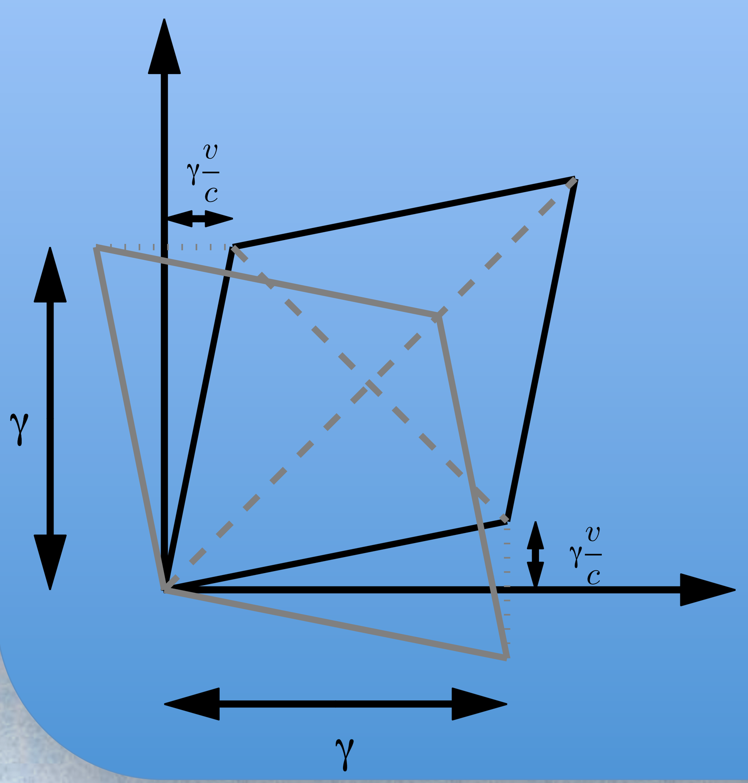
INTRODUCTION

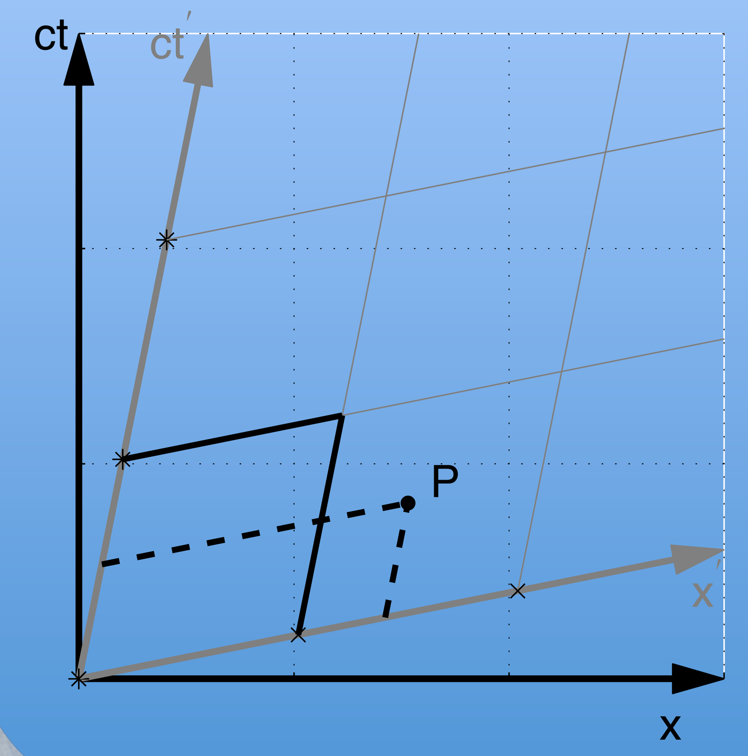
We present a graphical method that can be used to learn/teach Einstein's special theory of relativity. In our study, we adapted and modernized the Minkowski diagram in a way that would help current college students progress from Galilean transformations (based on Newton's Three Laws of Motion) to the kinematics of relativity and other key results. This study has revealed new ways to apply Minkowski diagrams to well known problems in relativity. Here, we will introduce our graphical technique and apply it to the resolution of three paradoxes.

UNIT CELL



Using the two dashed lines, which represent right- and left-going light pulses, we find how a moving observer's space and time axes deform. According to Einstein's postulate-the speed of light is the same in any reference frame-light must travel one x unit in one time unit. So, the dark (light) parallelogram represents an allowed unit cell of an observer moving to the right (left).

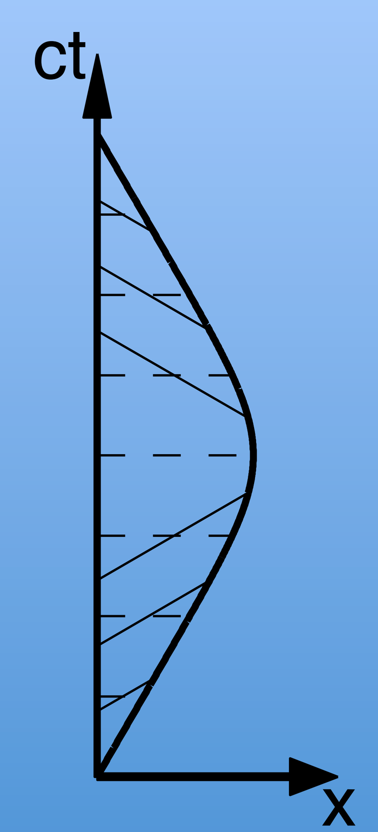
SPACE-TIME REPRESENTATION



A point P is represented differently in two reference frames. In observer 1's reference frame (dark axes), x and ct are simply the Cartesian projections of P. Observer 2's x' and ct' coordinates are obtained by the projection indicated with the dashed lines.

TWIN PARADOX

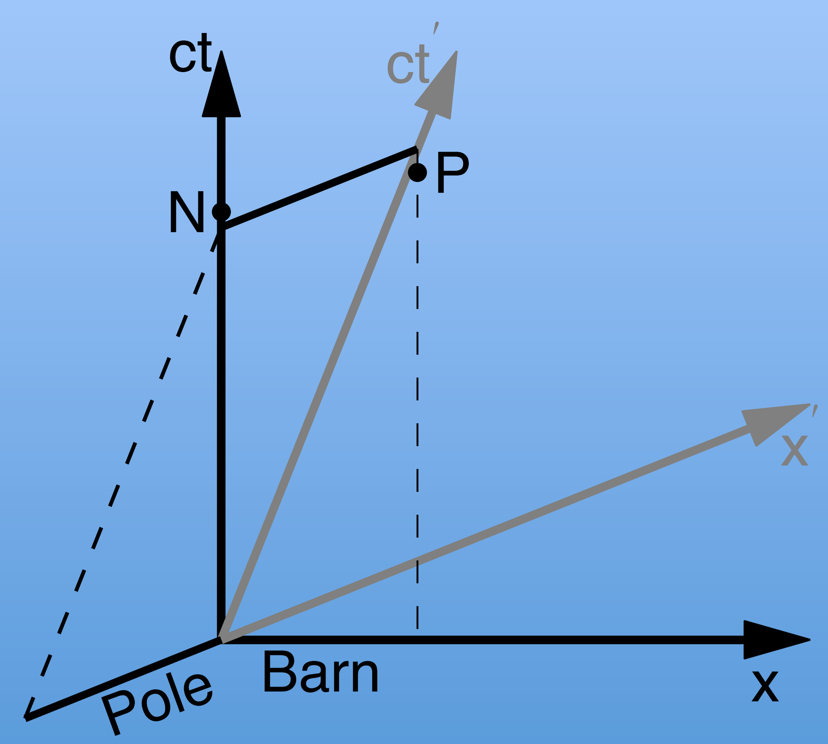
In the twin paradox, one twin stays on the earth while the other races to a distant point in space at a significant fraction of the speed of light. The latter twin then quickly turn around and returns to the earth at the same speed. When the twins reunite, the earth-bound twin has aged more than the astronaut twin. However, the motion of the astronaut twin as observed by the earth-bound twin is exactly the same as the motion of the earth-bound twin as observed by the astronaut twin. Therefore, how could there be an asymmetry in their aging?



The light solid- and dashed-lines are the astronaut and earth twin's lines of simultaneity. During most of the out-bound and in-bound trip, each twin observes the other's time progressing at a slower rate than his own. But notice that during year 3 and 4 on the astronaut's calendar (when he turns around), he observes the earth twin aging more than three years!

BARN-POLE PARADOX

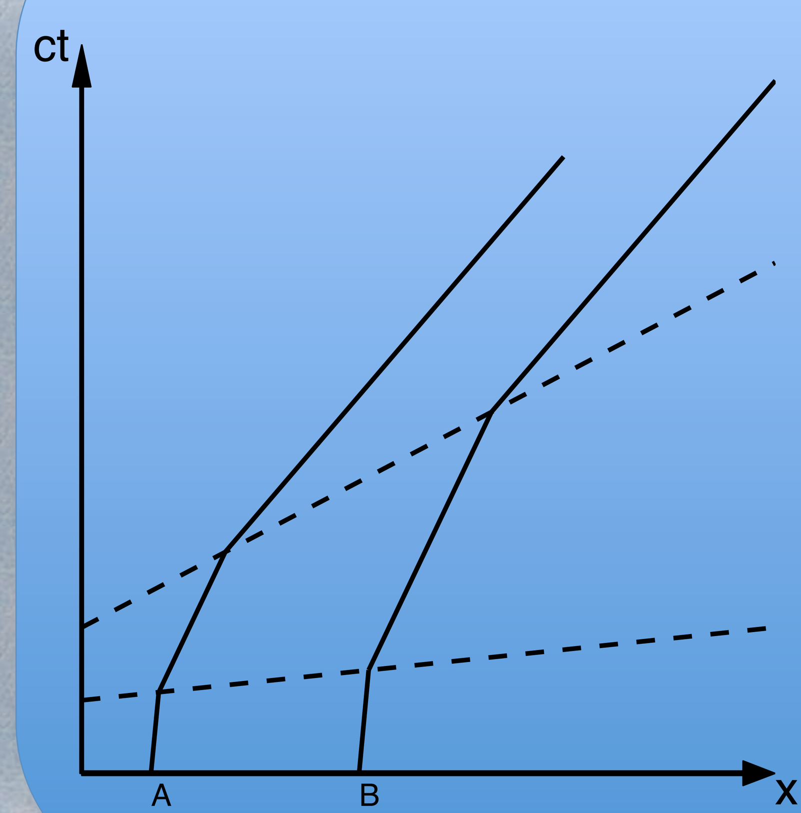
The Flash with a pole decided to run through a barn slightly shorter than the length of the pole. Initially, the front door of the barn is open and the backdoor closed. In the barn's reference frame, the Flash is moving at such high speed that the pole contracts and fits within the barn. When the pole is fully inside the barn, the front door closes, and the back door opens at the exact same time. So it will not bump into the backdoor. On the other hand, the Flash observes the barn to contract since he sees the barn moving towards him at the same speed, and he thinks he will bump into the backdoor. Who is right?



In the barn's reference frame, the pole will contract and will fit into the barn. In the Flash's reference frame, the backdoor opens (event P) *before* the front door closes (event N) so he crosses the barn without bumping into the backdoor.

BELL'S PARADOX

Two spaceships tied together with a rope at rest relative to the earth. The rope is stretched to its limit and will break if stretched further. Then, the two spaceships accelerate to reach cruising speed. The pilots believe that the rope will not snap as long as they accelerate *in unison*. An earth-bound observer objects. He believes once the two spaceship reaches higher speed, the length of the rope contracts while the two spaceship are still at the same distance from each other due to the synchronized acceleration. Who is right?



Neither is wrong, just the statement "synchronized acceleration" cannot apply to both the spaceships' and the observer's reference frame. In the case where the spaceships accelerate simultaneously in their reference frame, it appears to the earth observer that spaceship A start accelerating before B, and the two spaceships become closer as the rope contracts.