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Decomposing Complete Graphs into a Graph Pair of Order 6

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Decomposing complete graphs into a graph pair of order 6

**Purpose:**
Decomposing $K_n$ into a particular graph pair of order 6.

**Definition:**
Graph: A graph $G$ is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge two vertices called its end points.

A complete graph is a graph in which each pair of graph vertices is connected by an edge.

The complement of a graph $G$ is the graph with the same vertex set by whose edge set consists of the edges not present in $G$.

A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

A graph pair of order $n$ is a pair of connected graphs on $n$ vertices with no isolated vertex whose union is $K_n$. In this case, we will use $C_6$ and its complement to decompose $K_n$.

**A Proof**
By simple algebra, necessary conditions for a multidecomposition of $K_n$ into $C_6$ and $\hat{C}_6$ are $n=0, 1, 3, 4 \mod 6$. Then, we want to show that these conditions are sufficient by constructing a multidecomposition in each case.

1) Show that $K_n$ can be decomposed into $C_6$ and its complement if $n=0 \mod 6$.

In this case, $K_n$ can be seen as the union of many $K_6$ s connected with $K_{6,6}$ s. $K_6$ s can be decomposed into $C_6$ s and their complements (one copy in each). We need to show that $K_{6,6}$ s can also be decomposed into $C_6$ s or its complement. By Sotteau's theorem, $K_{6,6}$ can be decomposed into $C_6$ s. Hence, since $K_n$ can be decomposed into $C_6$ and its complement when $n=0 \mod 6$, the decomposition exists when $n=0 \mod 6$.

2) Show that $K_n$ can be decomposed into $C_6$ and its complement when $n=3 \mod 6$.

First, we can take a look at some small examples of $K_n$ when $n=3 \mod 6$. For example, are we able to show that $K_9$ can be decomposed into graph pair of order 6?

By edge condition, since $K_9$ must have at least one complement, there are $36-9=27$ edges left. After doing some simple algebra, I find that the decomposition can only exist if there are 3 more $C_6$ s and one $\hat{C}_6$ in $K_9$.

Then, we need to see whether the decomposition above exists given the degree condition. Each vertex of $K_9$ must have degree of 8. Since we have already removed one $C_6$ complement, 6 vertices have degree of 5 left. However, since a $C_6$ removes 2 degrees from each vertex and $5/2$ is not an integer, it is not true that $K_9$ can be decomposed into graph pair of order 6 since some vertices only have degree of 5 left.

**Current Result**
$K_n$ can be decomposed into graph pair consisting of $C_6$ s and their complements if $n$ is 0 mod 6. Also, $K_9$ can not be decomposed in that way.

**Future Study:**
I will try to test the sufficient conditions for the decomposition of $K_n$ if $n=1, 3$ and 4 mod 6.